微積分聯合教學 (物理、化材、地科、大氣)　期末考試題卷　**Jan 13, 2005**

考試時間 100 分鐘，請盡量依照題號順序將答案寫在答案卷上，不必抄題。試題卷有三面，共 8 大題。答案卷務必記得寫學號、姓名，試題卷不必繳回。考試開始 20 分鐘後不得入場，開始 40 分鐘前不得離場。爲維持機會之平等，考試期間禁止使用字典、計算機及任何通訊器材。

1. (*20* points)　是非題，請答 **T** (True) 或 **F** (False)

1.1 $\dfrac{d}{dx}2^x = x\,2^{x-1}$

1.2 $\displaystyle\int_1^2 \frac{1}{x}\,dx + \int_1^3 \frac{1}{x}\,dx = \int_1^6 \frac{1}{x}\,dx$

1.3 $\sinh^2 x + \cosh^2 x = 1$

1.4 $\displaystyle\lim_{x\to 3}\frac{x-3}{x^2-3} = \lim_{x\to 3}\frac{1}{2x} = \frac{1}{6}$

1.5 $\displaystyle\int_{-2}^2 \frac{1}{x}\,dx = [\ln|x|]_{-2}^2 = \ln 2 - \ln 2 = 0$

1.6 If $a_n \geq 0$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges.

1.7 $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n+1}$ converges.

1.8 $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}$ converges conditionally.

1.9 $\displaystyle\int_2^{\infty}\frac{1}{x^2}\,dx = \frac{1}{2}$

1.10 If the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\,a_n$ is convergent, then

$$\sum_{n=1}^{\infty}(-1)^{n+1}\,a_n = \sum_{n=1}^{\infty} a_{2n-1} - \sum_{n=1}^{\infty} a_{2n}$$

2. (*20* points)　選擇題，皆單選，請用大寫字母 **A**, **B**, **C** 或 **D** 答題

2.1 $\displaystyle\lim_{x\to 0}(1+2x)^{\frac{1}{x}} =$

(A) $2e$　(B) $e/2$　(C) e^2　(D) $\sqrt{e}$

(後面還有)

2.2 $\displaystyle\int_2^\infty \frac{1}{x^2}\,dx$

(A) $=0$ (B) $=\frac{1}{2}$ (C) $=1$ (D) is divergent

2.3 $\displaystyle\int_0^\infty xe^{-x}\,dx$

(A) $=0$ (B) $=\frac{1}{2}$ (C) $=1$ (D) is divergent

2.4 Let

$$f(x)=\ln\frac{(1+x)(2+x)^2(3+x)^3(4+x)^4}{(1-x)(2-x)^2(3-x)^3(4-x)^4}$$

then $f'(0)=$

(A) 2 (B) 4 (C) 6 (D) 8

2.5 Which of the following is *not* correct?

(A) $\displaystyle\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$

(B) $\displaystyle\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$

(C) $\displaystyle\frac{d}{dx}\sec^{-1}x=\frac{1}{x\sqrt{1+x^2}}$

(D) $\sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{2}$

2.6 If the interval of convergence (收斂區間) for a power series $\sum a_n(x-1)^n$ is $(-1,3]$, then its radius of convergence (收斂半徑) is

(A) 0 (B) 1 (C) 2 (D) 3

2.7 The *Taylor's Theorem* states that if $f(x)$ is differentiable through order $n+1$ in an open interval I containing a, then for each $x\in I$

$$f(x)=f(a)+f'(a)(x-a)+\cdots+\frac{f^{(n)}(a)}{n!}(x-a)^n+R_n(x)$$

where the *remainder*

(A) $R_n(x)=\dfrac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1}$

(B) $R_n(x)=\dfrac{f^{(n+1)}(a)}{(n+1)!}(x-c)^{n+1}$ for some number c between a and x

(C) $R_n(x)=\dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ for some number c between a and x

(D) $R_n(x)=\dfrac{f^{(n+1)}(c)}{(n+1)!}(x-c)^{n+1}$ for some number c between a and x

(後面還有)

2.8 Let

$$A(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

then what is $A(\pi)$?

(A) -1 (B) 0 (C) e^{π} (D) $e^{-\pi}$

2.9 For what values of x will the Maclaurin series generated by $\dfrac{1}{1+2x}$ converge to $\dfrac{1}{1+2x}$?

(A) $-\infty < x < \infty$ (B) $-\frac{1}{2} < x < \frac{1}{2}$ (C) $0 < x < 1$ (D) $-1 < x < 1$

2.10 考慮一個數列的收斂與發散，則下列敘述何者為真？

(A) 有界必收斂 (B) 收斂必有界 (C) 發散必無界 (D) 以上皆是

3. (*10* points) Determine if the following series is convergent or divergent. Give reasons and show your work.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^3-1}}$$

4. (*10* points) If $p > 1$, determine if the following series is convergent or divergent. Give reasons and show your work.

$$\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^p}$$

5. (*10* points) Evaluate the indefinite integral

$$\int e^{\theta}\,\sin\theta\,d\theta$$

6. (*10* points) Evaluate the indefinite integral

$$\int \frac{x+4}{x^2+5x-6}\,dx$$

7. (*10* points) Evaluate the indefinite integral

$$\int \frac{dx}{4x^2+4x+2}$$

8. (*10* points) Solve the differential equation

$$\begin{cases} \dfrac{dy}{dx} = x^2\sqrt{y} & (x > 0 \text{ and } y > 0) \\ y(0) = 0 \end{cases}$$