考試時間 100 分鐘,請盡量依照題號順序將答案寫在答案卷上,不必抄題。試題卷有四面,共 8 大題。答案卷務必記得寫學號、姓名,試題卷不必繳回。考試開始 20 分鐘後不得入場,開始 40 分鐘前不得離場。為維持機會之平等,考試期間禁止使用字典、計算機及任何通訊器材。

1. (*20* points)　是非題,請答 **T** (True) 或 **F** (False)

1.1　$f(x) = \sqrt{1 + |x|}$ is an even function.

1.2　$\displaystyle\lim_{x \to 0} \frac{x}{|x|} = 1$.

1.3　If $f(x) = g(x)$ for all real numbers x except for $x = 0$, then

$$\lim_{x \to 0} f(x) = L \quad \Rightarrow \quad \lim_{x \to 0} g(x) = L$$

1.4　If $f(x)$ is continuous at $x = 0$ then it is differentiable at $x = 0$.

1.5　The average value of $|x|$ on $[-1, 1]$ is $\frac{1}{2}$.

1.6　Let $f(x)$ be a continuous function on $[a, b]$ and $F(x) = \displaystyle\int_a^x f(t)\, dt$ for $x \in [a, b]$, then $F(x)$ is continuous on $[a, b]$.

1.7　Since $\sec 0 = 1$ and $\sec \pi = -1$, so by the *intermediate value theorem* there is a number $\xi \in (0, \pi)$ such that $\sec \xi = 0$.

1.8　If $\displaystyle\lim_{x \to c} f(x) = 0$, then there must exist a number ξ such that $f(\xi) < 0.001$.

1.9　For any continuous function $f(x)$ there is a differentiable function $F(x)$ such that $F'(x) = f(x)$.

1.10　Let
$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & \text{for } x \neq 0, \\ 0, & \text{for } x = 0 \end{cases}$$

then $f(x)$ is differentiable for all x.

2. (*20* points)　選擇題，皆單選，請用大寫字母 **A**, **B**, **C** 或 **D** 答題

2.1 How many anti-derivatives does the function $\sin^2 x - 3x$ have?

(A) 0　　(B) 1　　(C) 2　　(D) infinitely many

2.2 Let $y = \sin\dfrac{x}{2}\cos\dfrac{x}{2}$, then $\dfrac{d^{20}y}{dx^{20}} =$

(A) $\dfrac{\sin x}{2}$　　(B) $-\dfrac{\sin x}{2}$　　(C) $\dfrac{\cos x}{2}$　　(D) $-\dfrac{\cos x}{2}$

2.3 If $\lim\limits_{x\to 0}\dfrac{g(x)-1}{x} = 4$, then $\lim\limits_{x\to 0} g(x) =$

(A) 0　　(B) 1　　(C) 4　　(D) ∞

2.4 Which of the following equations is wrong?

(A) $\displaystyle\int 2\sin x \cos x \, dx = \sin^2 x + C$

(B) $\displaystyle\int 2\sin x \cos x \, dx = -\cos^2 x + C$

(C) $\displaystyle\int x \sin x \, dx = \dfrac{x^2}{2}\sin x + C$

(D) $\displaystyle\int x \sin x \, dx = \sin x - x\cos x + C$

2.5 Let $f(x)$ be a continuous function on $[0,1]$ and $F(x) = \int_0^x f(t)\,dt$ for $x \in [0,1]$, which of the following statements is false?

(A) $F'(x) = \lim\limits_{h\to 0}\dfrac{1}{h}\displaystyle\int_x^{x+h} f(t)\,dt$ for $x \in (0,1)$.

(B) $F(x)$ is the solution of $\dfrac{dy}{dx} = f(x)$ with initial condition $y(0) = 1$.

(C) $\displaystyle\int_0^1 f(x)\,dx = F(1) - F(0)$.

(D) $F'(x) = f(x)$ for $x \in (0,1)$.

2.6 What is the slope of the curve $y^2 + x^2 = y^4 - 2x$ at the point $(-2,1)$?

(A) -1　　(B) 0　　(C) 1　　(D) 2

(後面還有)

2.7 What is the value of
$$\lim_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$$

(A) $-\frac{1}{2}$ (B) 0   (C) $\frac{1}{2}$ (D) ∞

2.8 If $f'(0) = 1$ then $\displaystyle\lim_{h \to 0} \frac{f(2h) - f(0)}{h} = ?$

(A) 0   (B) $\frac{1}{2}$ (C) 1   (D) 2

2.9 What is the maximum value of $|f''(x)|$ for the function $f(x) = x^3(3x^2 - 10)$ on the closed interval $[0,1]$?

(A) 0   (B) 7   (C) $\frac{40}{\sqrt{3}}$ (D) $20\sqrt{3}$

2.10 Which of the following statements is correct?

(A) If $f'(c) = 0$ then $f(c)$ is a local maximum

(B) If $f'(c) = 0$ then $f(c)$ is a local minimum

(C) If $f'(c) > 0$ then $f(c)$ is not a local maximum

(D) If $f'(c)$ does not exist, then $f(c)$ is not a local maximum

3. (*10* points)   Show that for any numbers a and b, the inequality

$$|\cos a - \cos b| \le |a - b|$$

is true.

4. (*10* points)   Show that

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{for } x \ne 0, \\ 0, & \text{otherwise} \end{cases}$$

is differentiable at $x = 0$ and find $f'(0)$.

5. (*10* points)   Choose a linearization of $f(x) = \sqrt[3]{x}$ with center $x = a$, where a is near but is not 8.5. Find such a center a at which the function and its derivative are easy to evaluate. State the linearization and the center.

<div align="center">(後面還有)</div>

6. (*10* points) The region enclosed by the x-axis and the parabola $y = 3x - x^2$ is revolved about the line $x = -1$ to generate the shape of a solid. Compute the volume of the solid.

7. (*10* points) Suppose that f is the differentiable function shown in the accompanying graph (附圖) and the position (位置) at time t seconds of a particle (粒子) moving along a coordinate axis (沿著一個坐標軸移動) is

$$s(t) = \int_0^t f(x)\,dx$$

[從課本剪貼一張圖]

Answer the following questions and give your reasons (說明理由):

   (a) What is the particle's velocity at time $t = 5$?

   (b) Is the acceleration of the particle at time $t = 5$ positive or negative?

   (c) Approximately when is the acceleration zero?

   (d) Is $s(9)$ positive or negative?

   (e) At what time between $0 \le t \le 9$ does $s(t)$ have its largest value?

8. (*10* points) Let $f(x)$ be an integrable function, show that

$$\int_0^\pi x\,f(\sin x)\,dx = \frac{\pi}{2} \int_0^\pi f(\sin x)\,dx$$

[Hint: Use the substitution $u = \pi - x$.]