微積分聯合教學 (物理、化材、地科、大氣)　期中考試題卷　**Apr 21, 2005**

考試時間 100 分鐘，請盡量依照題號順序將答案寫在答案卷上，不必抄題。試題卷有三面，共 8 大題。答案卷務必記得寫學號、姓名，試題卷不必繳回。考試開始 20 分鐘後不得入場，開始 40 分鐘前不得離場。爲維持機會之平等，考試期間禁止使用字典、計算機及任何通訊器材。

1. (*20* points)　是非題，請答 **T** (True) 或 **F** (False)

1.1 If $f(x,y)$ has partial derivatives $f_x(x_0,y_0)$ and $f_y(x_0,y_0)$, then it is differentiable at (x_0,y_0).

1.2 If $f(x,y)$ is differentiable at (x_0,y_0), then it is continuous at (x_0,y_0).

1.3 Velocity function is a continuous function of time.

1.4 $(\mathbf{i}-\mathbf{k})\times(\mathbf{j}+\mathbf{k})=\mathbf{i}-\mathbf{j}+\mathbf{k}$

1.5 $\dfrac{\partial}{\partial x}\dfrac{1}{1-x-y}=\dfrac{1}{(1-x-y)^2}$

1.6 In polar coordinates, the point $(2,\frac{3\pi}{4})$ lies on the curve $r=2\sin 2\theta$.

1.7 Let $\mathbf{u}$ and $\mathbf{v}$ denote two vectors, then $|\mathbf{u}\cdot\mathbf{v}|^2+|\mathbf{u}\times\mathbf{v}|^2=|\mathbf{u}|^2\,|\mathbf{v}|^2$.

1.8 Let $\mathbf{u}$ and $\mathbf{v}$ denote two vectors, then $\mathbf{u}\times\mathbf{v}+\mathbf{v}\times\mathbf{u}=\mathbf{0}$.

1.9 Velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of a moving particle (運動中的粒子) determines the curvature of its path (決定其軌跡的曲率).

1.10 Suppose that $f(x,y)$ and its first and second derivatives are continuous throughout a disk centered at (x_0,y_0) and that $f_x(x_0,y_0)=f_y(x_0,y_0)=0$. Then the "second derivative test" says that f has a local minimum at (x_0,y_0) if $f_{xx}f_{yy}-f_{xy}^2>0$ at (x_0,y_0).

2. (*20* points)　選擇題，皆單選，請用大寫字母 **A**, **B**, **C** 或 **D** 答題

2.1 In Cartesian coordinates, the graph of $x=1$ could **not** be

(A) a point (B) a line (C) a plane (D) a circle

2.2 In polar coordinates, how many horizontal tangent lines (水平切線) are there for the curve $r=-1+\sin\theta$ for $\theta\in[0,2\pi]$?

(A) 3　(B) 2　(C) 1　(D) 0

(後面還有)

2.3 In polar coordinates, how many points of intersection are there for the curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$?

(A) 3 (B) 2 (C) 1 (D) 0

2.4 In what direction does $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decrease most rapidly at the point $(1, 1)$?

(A) $\mathbf{i} + \mathbf{j}$ (B) $-\mathbf{i} - \mathbf{j}$ (C) $-\mathbf{i} + \mathbf{j}$ (D) $\mathbf{i} - \mathbf{j}$

2.5 Which of the following is the vector projection of $\mathbf{F} = 5\mathbf{i} + 2\mathbf{j}$ onto $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$?

(A) $-\frac{1}{10}\mathbf{i} - \frac{3}{10}\mathbf{j}$ (B) $\frac{1}{10}\mathbf{i} + \frac{3}{10}\mathbf{j}$ (C) $-\frac{1}{10}\mathbf{i} + \frac{3}{10}\mathbf{j}$ (D) $\frac{1}{10}\mathbf{i} - \frac{3}{10}\mathbf{j}$

2.6 What is the volume of the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{8} = 2$?

(A) $\frac{128\pi}{3}$ (B) $\frac{64\pi}{3}$ (C) $\frac{32\pi}{3}$ (D) $\frac{16\pi}{3}$

2.7 What is the equation of the tangent plane of the surface $x^2 - y^2 - z = 0$ at the point $(2, 1, 3)$?

(A) $2(x - 2) - (y - 1) + 4(z - 3) = 0$

(B) $4(x - 2) - 2(y - 1) - (z - 3) = 0$

(C) $4(x - 2) + 2(y - 1) - (z - 3) = 0$

(D) $2(x - 2) + (y - 1) + 4(z - 3) = 0$

2.8 In order to maximize or minimize $f(x, y, z)$ subject to $g(x, y, z) = 0$, ∇f and ∇g should be calculated for the method of Lagrange multipliers. Let (x_0, y_0, z_0) denote the solution, then

(A) the angle between $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ must be 0

(B) the angle between $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ must be π

(C) the dot product of $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ is 0

(D) the cross product of $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ is 0

2.9 If

$$w = x^2 + y^2 + z^2, \quad z^3 - xy + yz + y^3 = 1$$

where x and y are the independent variables, what is the partial derivative $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$?

(A) 3 (B) 2 (C) 1 (D) 0

(後面還有)

2.10 Which in the following is the Taylor polynomial of second degree for $\dfrac{1}{1-x-y}$ centered at the origin?

(A) $1+(x+y)+(2x^2+4xy+2y^2)$

(B) $1+(x+y)+\dfrac{1}{2}(2x^2+4xy+2y^2)$

(C) $1+(x+y)+(2x^2+2y^2)$

(D) $1+(x+y)+\dfrac{1}{2}(2x^2+2y^2)$

3. (*10* points) In polar coordinates, find the area of the region that lies inside the cardioid curve (心臟線) $r=1+\cos\theta$ and outside the circle $r=\cos\theta$.

4. (*10* points) Let

$$f(x,y)=\begin{cases}\dfrac{2xy^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0),\\ 0, & \text{otherwise.}\end{cases}$$

Is $f(x,y)$ continuous everywhere? Explain your answer and show your reasons.

5. (*10* points) Write $\mathbf{a}$ for the vector valued function

$$\mathbf{r}(t)=(e^t\cos t)\,\mathbf{i}+(e^t\sin t)\,\mathbf{j}+\sqrt{2}e^t\,\mathbf{k}$$

in the form $\mathbf{a}=a_T\mathbf{T}+a_N\mathbf{N}$ at $t=0$ without finding $\mathbf{T}$ and $\mathbf{N}$.

6. (*10* points) Find the linearization $L(x,y)$ of the function

$$f(x,y)=x^2-xy+\frac{1}{2}y^2+3$$

at the point $(3,2)$. Then find an upper bound for the error in the approximation $f(x,y)\approx L(x,y)$ over the rectangle R: $|x-3|\leq 0.1$, $|y-2|\leq 0.1$.

7. (*10* points) Find all the local maxima, local minima and saddle points of

$$f(x,y)=9x^3+\frac{1}{3}y^3-4xy$$

8. (*10* points) Find the maximum and minimum values of

$$f(x,y,z)=x-2y+5z$$

on the sphere $x^2+y^2+z^2=30$.