1. (20 points) Assume that, please answer T (True) or F (False)

1.1 If \( f(x, y) \) has partial derivatives \( f_x(x_0, y_0) \) and \( f_y(x_0, y_0) \), then it is differentiable at \( (x_0, y_0) \).

1.2 If \( f(x, y) \) is differentiable at \( (x_0, y_0) \), then it is continuous at \( (x_0, y_0) \).

1.3 Velocity function is a continuous function of time.

1.4 \((i - k) \times (j + k) = i - j + k\)

1.5 \( \frac{\partial}{\partial x} \frac{1}{1 - x - y} = \frac{1}{(1 - x - y)^2} \)

1.6 In polar coordinates, the point \( (2, \frac{3\pi}{4}) \) lies on the curve \( r = 2\sin 2\theta \).

1.7 Let \( \mathbf{u} \) and \( \mathbf{v} \) denote two vectors, then \( |\mathbf{u} \cdot \mathbf{v}|^2 + |\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2 |\mathbf{v}|^2 \).

1.8 Let \( \mathbf{u} \) and \( \mathbf{v} \) denote two vectors, then \( \mathbf{u} \times \mathbf{v} + \mathbf{v} \times \mathbf{u} = \mathbf{0} \).

1.9 Velocity \( \mathbf{v} \) and acceleration \( \mathbf{a} \) of a moving particle (運動中的粒子) determines the curvature of its path (決定其軌跡的曲率).

1.10 Suppose that \( f(x, y) \) and its first and second derivatives are continuous throughout a disk centered at \( (x_0, y_0) \) and that \( f_x(x_0, y_0) = f_y(x_0, y_0) = 0 \). Then the “second derivative test” says that \( f \) has a local minimum at \( (x_0, y_0) \) if \( f_{xx}f_{yy} - f_{xy}^2 > 0 \) at \( (x_0, y_0) \).

2. (20 points) Select one, please answer A, B, C or D answer

2.1 In Cartesian coordinates, the graph of \( x = 1 \) could not be

(\( A \) a point  \( B \) a line  \( C \) a plane  \( D \) a circle)

2.2 In polar coordinates, how many horizontal tangent lines (水平切線) are there for the curve \( r = -1 + \sin \theta \) for \( \theta \in [0, 2\pi] \)?

(\( A \) 3  \( B \) 2  \( C \) 1  \( D \) 0)

(後面還有)
2.3 In polar coordinates, how many points of intersection are there for the curves $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$?

(A) 3  (B) 2  (C) 1  (D) 0

2.4 In what direction does $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decrease most rapidly at the point $(1, 1)$?

(A) $i + j$  (B) $-i - j$  (C) $-i + j$  (D) $i - j$

2.5 Which of the following is the vector projection of $\mathbf{F} = 5\mathbf{i} + 2\mathbf{j}$ onto $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$?

(A) $-\frac{1}{10} \mathbf{i} - \frac{3}{10} \mathbf{j}$  (B) $\frac{1}{10} \mathbf{i} + \frac{3}{10} \mathbf{j}$  (C) $-\frac{1}{10} \mathbf{i} + \frac{3}{10} \mathbf{j}$  (D) $\frac{1}{10} \mathbf{i} - \frac{3}{10} \mathbf{j}$

2.6 What is the volume of the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{8} = 2$?

(A) $\frac{128\pi}{3}$  (B) $\frac{64\pi}{3}$  (C) $\frac{32\pi}{3}$  (D) $\frac{16\pi}{3}$

2.7 What is the equation of the tangent plane of the surface $x^2 - y^2 - z = 0$ at the point $(2, 1, 3)$?

(A) $2(x - 2) - (y - 1) + 4(z - 3) = 0$
(B) $4(x - 2) - 2(y - 1) - (z - 3) = 0$
(C) $4(x - 2) + 2(y - 1) - (z - 3) = 0$
(D) $2(x - 2) + (y - 1) + 4(z - 3) = 0$

2.8 In order to maximize or minimize $f(x, y, z)$ subject to $g(x, y, z) = 0$, $\nabla f$ and $\nabla g$ should be calculated for the method of Lagrange multipliers. Let $(x_0, y_0, z_0)$ denote the solution, then

(A) the angle between $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ must be 0
(B) the angle between $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ must be $\pi$
(C) the dot product of $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ is 0
(D) the cross product of $\nabla f(x_0, y_0, z_0)$ and $\nabla g(x_0, y_0, z_0)$ is 0

2.9 If

$$w = x^2 + y^2 + z^2, \quad z^3 - xy + yz + y^3 = 1$$

where $x$ and $y$ are the independent variables, what is the partial derivative $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$?

(A) 3  (B) 2  (C) 1  (D) 0

(後面還有)
2.10 Which in the following is the Taylor polynomial of second degree for \( \frac{1}{1 - x - y} \) centered at the origin?

(A) \( 1 + (x + y) + (2x^2 + 4xy + 2y^2) \)

(B) \( 1 + (x + y) + \frac{1}{2}(2x^2 + 4xy + 2y^2) \)

(C) \( 1 + (x + y) + (2x^2 + 2y^2) \)

(D) \( 1 + (x + y) + \frac{1}{2}(2x^2 + 2y^2) \)

3. (10 points) In polar coordinates, find the area of the region that lies inside the cardioid curve \( r = 1 + \cos \theta \) and outside the circle \( r = \cos \theta \).

4. (10 points) Let

\[
f(x, y) = \begin{cases} 
\frac{2xy^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\
0, & \text{otherwise.}
\end{cases}
\]

Is \( f(x, y) \) continuous everywhere? Explain your answer and show your reasons.

5. (10 points) Write \( a \) for the vector valued function

\[
\mathbf{r}(t) = (e^t \cos t) \mathbf{i} + (e^t \sin t) \mathbf{j} + \sqrt{2}e^t \mathbf{k}
\]

in the form \( \mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N} \) at \( t = 0 \) without finding \( \mathbf{T} \) and \( \mathbf{N} \).

6. (10 points) Find the linearization \( L(x, y) \) of the function

\[
f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3
\]

at the point \((3, 2)\). Then find an upper bound for the error in the approximation \( f(x, y) \approx L(x, y) \) over the rectangle \( R: |x - 3| \leq 0.1, |y - 2| \leq 0.1 \).

7. (10 points) Find all the local maxima, local minima and saddle points of

\[
f(x, y) = 9x^3 + \frac{1}{3}y^3 - 4xy
\]

8. (10 points) Find the maximum and minimum values of

\[
f(x, y, z) = x - 2y + 5z
\]
on the sphere \( x^2 + y^2 + z^2 = 30 \).