

考試時間 100 分鐘，請盡量依照題號順序將答案寫在答案卷上，不必抄題。  
試題卷有兩張共三面，共 10 大題。答案卷務必記得寫學號、姓名，試題卷  
不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。為維  
持機會之平等，考試期間禁止使用字典、計算機及任何通訊器材。

1. (10 points) 是非題，請答 **T** (True) 或 **F** (False)

1.1  $\lim_{x \rightarrow a} \frac{\lfloor x \rfloor}{x} \neq 1$  for all  $a \neq 0$ .

1.2 If  $f$  is an odd function, then  $\lim_{x \rightarrow 0} f(x) = 0$ .

1.3 If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = L$ ,  $\lim_{x \rightarrow a} g(x) = M$ , then  $\lim_{x \rightarrow a} f(x) = LM$ .

1.4 Let  $L$  and  $x_0$  be real numbers and  $g(x)$  be defined on  $(x_0 - 1, x_0 + 1)$ . Then

$$\lim_{x \rightarrow x_0} g(x) = L \text{ if and only if } \lim_{h \rightarrow 0} g(x_0 + h) = L.$$

1.5 The function  $y = \sqrt[3]{x}$  defined on the real numbers is differentiable at  $x = 0$ .

1.6 If  $f'(c) = 0$ , then  $f(c)$  must be an extreme value.

1.7 If  $f''(c) = 0$ , then  $(c, f(c))$  must be a point of inflection.

1.8 If  $f(x)$  is continuous at  $x = c$ , then  $f(x)$  is differentiable at  $x = c$ .

1.9  $x^{1/3} = \frac{x}{|x|} |x|^{1/3}$  for  $x < 0$ .

1.10 For any numbers  $a$  and  $b$ , the inequality  $|\cos b - \cos a| \leq |b - a|$  is true.

2. (10 points) 選擇題，皆單選，請用大寫字母 **A**, **B**, **C** 或 **D** 答題

2.1  $\lim_{x \rightarrow 5^-} \frac{3x^2}{x^2 - 25} =$

(A)  $-\infty$  (B)  $+\infty$  (C) 3 (D) 0

2.2  $\lim_{\theta \rightarrow 0} \sin\left(\frac{\pi\theta}{\sin\theta}\right) =$

(A) does not exist (B) -1 (C) 1 (D) 0

(背面還有)

2.3 Let  $x = \sec^2 t - 1$  and  $y = \tan t$ . Near  $t = -\pi/4$ , is  $y$  a differentiable function of  $x$ ?

- (A) No, it isn't.
- (B) Yes,  $y = \sqrt{x}$ .
- (C) Yes,  $y = -\sqrt{x}$ .
- (D) Yes,  $\frac{dy}{dx}|_{x=1} = -4$ .

2.4 If  $f(x)$  is differentiable at  $x = a$ , then which of the following statements is not true?

- (A)  $(f(x))^5$  is differentiable at  $x = a$ .
- (B)  $\sqrt{f(x)}$  is differentiable at  $x = a$ .
- (C)  $5f(x)$  is differentiable at  $x = a$ .
- (D)  $f(x) + 5$  is differentiable at  $x = a$ .

2.5 If  $f(x) = x(x - 1)(x - 2)$ , then

- (A)  $f(x)$  is increasing on  $(0, 1)$ .
- (B)  $f(x)$  is decreasing on  $(0, 1)$ .
- (C)  $f(x)$  is concave up on  $(1, 2)$ .
- (D)  $f(x)$  is concave down on  $(1, 2)$ .

3. (15 points) Use the graphing strategy to sketch the graph of

$$f(x) = \frac{1 + x^2}{(x + 1)^2}.$$

4. (5 points) If  $f'(0) = 1$ , find  $\lim_{h \rightarrow 0} \frac{f(2h) - f(-h)}{h}$ .

5. (10 points) Explain why the equation  $\cos x = x$  has at least one solution.

6. (10 points) Find the linearization of  $f(x) = (\cos x)^{-1/5}$  at  $x = \pi/4$ .

7. (10 points) Use implicit differentiation to find  $dy/dx$  if

$$y^2 \cos\left(\frac{1}{y}\right) = 2x + 2y.$$

8. (10 points) Find

$$\frac{d^{20}}{dx^{20}} \left( \cos \frac{x}{2} \right) \left( \sin \frac{x}{2} \right).$$

9. (10 points) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 5.

10. (10 points) Find a line which touches  $x^2 + y^2 + 4x - 6y - 3 = 0$  at only one point  $(2, 3)$ .