

考試時間 100 分鐘，請盡量依照題號順序將答案寫在答案卷上，題號靠左排列，不必抄題。試題卷共有兩張，四面，9 大題。答案卷務必記得寫學號、姓名，試題卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。為維持機會之平等，考試期間禁止使用字典、計算機及任何通訊器材。監試人員不回答任何關於考題的疑問。

1. (20 points) 是非題，請答 **T** (True) 或 **F** (False)

1.1 The following identity is correct:

$$\lim_{x \rightarrow 3} \frac{x-3}{x^2-3} = \lim_{x \rightarrow 3} \frac{1}{2x} = \frac{1}{6}$$

1.2 If we use Newton's method to find decimal approximations to $\sqrt{2}$, then we can use the formula $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$ with the starting value $x_0 = 2$.

1.3 If $p(x)$ is a polynomial function, then $p(x)$ has exactly one antiderivative whose graph contains the origin.

1.4 $\int_{-1}^1 \sqrt{x^2 - x^4} dx = \int_{-1}^1 x\sqrt{1 - x^2} dx.$

1.5 If $f(x) \in [-1, 1]$ is an even continuous function, then $\int_{-1}^1 f(x) dx = 0.$

1.6 The average value of $|x|$ on $[-1, 1]$ is $\frac{1}{2}.$

1.7 Let $f(x)$ be a continuous function on $[a, b]$ and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$, then $F(x)$ is differentiable on $(a, b).$

1.8 Let $f(x)$ be a continuous function on $[a, b]$ and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$, then $F(x)$ is continuous on $[a, b].$

1.9 Let $f'(x) = h'(x)$ and $g'(x) = k'(x)$ with $f(0) = h(0) + 2$ and $g(0) = k(0) + 2.$ If the area of the region bounded by the graphs of f and g is 1, then the area of the region bounded by the graphs of h and k is also 1.

1.10 If $f(x) = x - 1, 0 \leq x \leq 3,$ then the area of the surface generated by revolving the curve $y = f(x)$ about the x -axis is $S = \int_0^3 2\sqrt{2}\pi(x - 1) dx.$

2. (20 points) 選擇題，皆單選，請用大寫字母 **A, B, C** 或 **D** 答題

2.1 What is the value of

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$$

- (A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) ∞ .

2.2 Which of the following equations is wrong?

(A) $\int \tan \theta \sec^2 \theta d\theta = \frac{\sec^3 \theta}{3} + C$

(B) $\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \tan^2 \theta + C$

(C) $\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2} \sec^2 \theta + C$

(D) $\int \tan \theta \sec^2 \theta d\theta = \frac{1}{2}(\tan^2 \theta - 1) + C$

2.3 Let $f(x)$ be a continuous function on $[a, b]$ and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$, which of the following statements is false?

(A) $F'(x) = \lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} f(t) dt$ for $x \in [a, b]$

(B) $F(x)$ is the solution of the initial value problem $\frac{dy}{dx} = f(x)$, $f(a) = 1$.

(C) $\int_a^b f(x) dx = F(b) - F(a)$.

(D) $F'(x) = f(x)$ for all $x \in (a, b)$.

2.4 What is the value of

$$\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1} dt.$$

- (A) 1 (B) 0 (C) -1 (D) ∞ .

2.5 The region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 2$. Which of the following is the area of the region R ?

- (A) $-\frac{9}{2}$ (B) $\frac{9}{2}$ (C) $\frac{10}{3}$ (D) $-\frac{10}{3}$.

2.6 The region enclosed by the x -axis and the parabola $y = 3x - x^2$ is revolved about the line $x = -1$ to generate the shape of a solid. Which of the following integrals computes the volume of the solid?

(A) $\int_0^3 \pi(3x - x^2)^2 dx$

(B) $\int_0^3 2\pi(x + 1)(3x - x^2)^2 dx$

(C) $\int_0^3 2\pi(3x - x^2)^2 dx$

(D) $\int_0^3 2\pi(x + 1)(3x - x^2) dx$

2.7 Which of the following is the volume of the torus (doughnut) generated by revolving a circular disk of radius a about an axis in its plane at a distance $b \geq a$ from its center?

(A) πba^2 (B) $2\pi ba^2$ (C) $2\pi^2 ba^2$ (D) $4\pi^2 ba$.

2.8 Which of the following is the length of the curve

$$x = 1 - t, \quad y = 2 + 3t, \quad -\frac{2}{3} \leq t \leq 1.$$

(A) $\sqrt{10}$ (B) $\frac{5\sqrt{10}}{3}$ (C) $\sqrt{10}\pi$ (D) 1.

2.9 Which of the following is the work required to compress a spring from its natural length of 1 ft to a length of 0.75 ft if the force constant is $k=16$ lb/ft?

(A) 1 ft-lb (B) 2 ft-lb (C) 4.5 ft-lb (D) 0.5 ft-lb.

2.10 A flat isosceles right triangular plate with base 6 ft and height 3 ft is submerged vertically, base up, 2 ft below the surface of a swimming pool. Which of the following integrals finds the force exerted by the water against one side of the plate. The weight-density of water is 62.4 lb/ft^3 .

(A) $\int_0^3 62.4(3 - y)y dy$ (B) $\int_0^3 62.4(3 - y) \cdot (2y) dy$

(C) $\int_0^3 62.4(5 - y)y dy$ (D) $\int_0^3 62.4(5 - y) \cdot (2y) dy$

3. (10 points) Find the following limit $\lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} \sqrt{t} - \tan \sqrt{t} dt}{\int_0^{x^2} \sqrt{t} - \sin \sqrt{t} dt}$.
4. (10 points) The region enclosed by the x -axis and the parabola $y = 2x - x^2$ is revolved about the line $y = 3$ to generate the shape of a solid. Compute the volume of the solid.
5. (10 points) Find the area of the surface generated by revolving $x = \frac{y^2}{4}$, $2 \leq y \leq 4$, about the x -axis.
6. (10 points) Find a function f that satisfies the conditions: $f''(x) = x^{-2/3}$, $f'(8) = 6$ and $f(0) = 0$.
7. (10 points) Find the area of the region bounded by the curves $y = x + 1$ and $y = x^2 - x + 1$.
8. (5/5 points) Evaluate the following integrals.
- (a) $\int \frac{dt}{t^2 \sqrt{3 - \frac{4}{t}}}$
- (b) $\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{1}{\sqrt{\theta}} (2 + \sin^2 \sqrt{\theta}) \cos \sqrt{\theta} d\theta$