Calculus　　Midterm Examination I　　April 3, 2007

考試時間 100 分鐘，配合、是非、填充題請在專用答案卷上作答，其他題目請盡量依照題號順序將答案寫在答案卷上，不必抄題。試題卷有兩張共三面。答案卷務必記得寫學號、姓名，試題卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

1. (*10* points)　配合題

Please see the following figures (a)-(h). Select the correct one for each of the following equations of surfaces.

1) $9x^2 + 4y^2 + 2z^2 = 36$　2) $z^2 + 4y^2 - 4x^2 = 4$　3) $x^2 + 2z^2 = 8$

4) $y^2 + z^2 = x^2$　5) $x = -y^2 - z^2$

2. (*10* points) 是非題，請答 **T** (True) 或 **F** (False)

2.1 If $\mathbf{u}$ and $\mathbf{v}$ are vectors, then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

2.2 The line $x = 1 + 2t$, $y = 2 + 5t$, $z = 3t$ is parallel to the plane $2x + y - 3z = 0$.

2.3 Let S be a circle of radius 3. Then the curvature of S is 3.

2.4 The limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2+y^2}$ does not exist.

2.5 If $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$, then f is continous at the origin (0, 0).

3. (*40* points) 填充題

3.1 The area of the triangle determined by $P(1,-2,2)$, $Q(2,0,-1)$, $R(0,2,1)$ is ______.

3.2 Let V be the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$. Then $V =$ __________. (The area of an ellipse with semiaxes a and b is πab.)

3.3 The angle between the planes $x + y = 1$ and $2x + y - 2z = 2$ is __________.

3.4 Let A be the minimum value of the function $f(x,y,z) = xy - z$ along the line $x = t-1, y = t-2, z = t+7$. Then $A =$ __________.

3.5 Let $w = x^2 + \frac{y}{x}$, $x = u - 2v + 1$, $y = 2u + v - 2$. If A is the value of $\frac{\partial w}{\partial v}$ at $(u,v) = (0,0)$, then $A =$ __________.

3.6 Define $f(0,0) =$ __________ in a way that extends f to be continuous at the origin, if

$$f(x,y) = \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right).$$

3.7 Let $L(x,y)$ be the linearization of the function $f(x,y) = x^2 + y^2 + 1$ at the point (1, 1). Then $L(x,y) =$ __________.

3.8 In what direction is the derivative of $f(x,y) = xy + y^2$ at $P(3,2)$ equal to zero?__________

計算題

4. (*10* points) Here $z = 4e^x \ln y$, $x = \ln(u \cos v)$, $y = u \sin v$.

a. Express $\partial z/\partial u$ and $\partial z/\partial v$ as functions of u and v.

b. Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the point $(2, \pi/4)$.

5. (*10* points) A glider is soaring upward along the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$.

a. How far does the glider travel along its path from $t = 0$ to $t = 2\pi$?

b. Find the unit tangent vector T, the curvature κ, and the principal unit normal vector N.

6. (*10* points) Find the equation of the tangent plane to the surface $z = x \cos y - ye^x$ at (0, 0, 0).

7. (*10* points)

a. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

b. In what directions does f change most rapidly at P_0, and what are the rates of change in these directions?