1. (10 points) 配合題

Please see the following figures (a)-(h). Select the correct one for each of the following equations of surfaces.

1) \(9x^2 + 4y^2 + 2z^2 = 36\)  
2) \(z^2 + 4y^2 - 4x^2 = 4\)  
3) \(x^2 + 2z^2 = 8\)  
4) \(y^2 + z^2 = x^2\)  
5) \(x = -y^2 - z^2\)
2. (10 points)是非題，請答 T (True) 或 F (False)

2.1 If \(\mathbf{u}\) and \(\mathbf{v}\) are vectors, then \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0\).

2.2 The line \(x = 1 + 2t, y = 2 + 5t, z = 3t\) is parallel to the plane \(2x + y - 3z = 0\).

2.3 Let \(S\) be a circle of radius 3. Then the curvature of \(S\) is 3.

2.4 The limit \(\lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2 + y^2}\) does not exist.

2.5 If \(f(x,y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}\), then \(f\) is continuous at the origin \((0,0)\).

3. (40 points)填充分題

3.1 The area of the triangle determined by \(P(1, -2, 2), Q(2, 0, -1), R(0, 2, 1)\) is ________.

3.2 Let \(V\) be the volume of the ellipsoid \(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\). Then \(V = \) ________.

(The area of an ellipse with semiaxes \(a\) and \(b\) is \(\pi ab\).)

3.3 The angle between the planes \(x + y = 1\) and \(2x + y - 2z = 2\) is ________.

3.4 Let \(A\) be the minimum value of the function \(f(x,y,z) = xy - z\) along the line \(x = t - 1, y = t - 2, z = t + 7\). Then \(A = \) ________.

3.5 Let \(w = x^2 + \frac{y}{x}, x = u - 2v + 1, y = 2u + v - 2\). If \(A\) is the value of \(\frac{\partial w}{\partial v}\) at \((u,v) = (0,0)\), then \(A = \) ________.

3.6 Define \(f(0,0) = \) ________ in a way that extends \(f\) to be continuous at the origin, if

\[
f(x,y) = \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2 + y^2}\right).
\]

3.7 Let \(L(x,y)\) be the linearization of the function \(f(x,y) = x^2 + y^2 + 1\) at the point \((1,1)\). Then \(L(x,y) = \) ________.

3.8 In what direction is the derivative of \(f(x,y) = xy + y^2\) at \(P(3,2)\) equal to zero? ________

2
4. (10 points) Here \( z = 4e^x \ln y, \) \( x = \ln(u \cos v), \) \( y = u \sin v. \)
   
a. Express \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) as functions of \( u \) and \( v. \)

b. Evaluate \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) at the point \( (2, \pi/4). \)

5. (10 points) A glider is soaring upward along the helix \( \mathbf{r}(t) = (\cos t)i + (\sin t)j + tk. \)
   
a. How far does the glider travel along its path from \( t = 0 \) to \( t = 2\pi? \)

b. Find the unit tangent vector \( T, \) the curvature \( \kappa, \) and the principal unit normal vector \( N. \)

6. (10 points) Find the equation of the tangent plane to the surface \( z = x \cos y - ye^x \) at \( (0, 0, 0). \)

7. (10 points)
   
a. Find the derivative of \( f(x, y, z) = x^3 - xy^2 - z \) at \( P_0(1, 1, 0) \) in the direction of \( \mathbf{v} = 2i - 3j + 6k. \)

b. In what directions does \( f \) change most rapidly at \( P_0, \) and what are the rates of change in these directions?