

考試時間 100 分鐘，是非、填充題請在專用答案卷上作答，其他題目請盡量依照題號順序將答案寫在答案卷上，不必抄題。試題卷有一張共兩面。答案卷務必記得寫學號、姓名，試題卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

1. (12 points) 是非題，請答 **T** (True) 或 **F** (False)

1.1 Continuous functions of two variables assume absolute extreme values on closed, bounded domains.

1.2 Given any two polar curves $r = f(\theta)$ and $r = g(\theta)$, if the point (r_0, θ_0) is an intersection point of the curves $r = f(\theta)$ and $r = g(\theta)$, then $r_0 = f(\theta_0)$ and $r_0 = g(\theta_0)$.

1.3 If the graph of a polar curve $r = f(\theta)$ passes through the origin at the value $\theta = \theta_0$, then the slope of the curve there is $\frac{df}{d\theta}|_{\theta_0} = \tan \theta_0$ provided the slope at $(0, \theta_0)$ exist.

1.4
$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx = \int_0^{\pi/2} \int_0^1 r^3 dr d\theta = \int_0^1 \int_0^{\pi/2} r^3 d\theta dr.$$

2. (40 points) 填充題

2.1 If $w = x^2 + y - z + \sin t$ and $x + y = t$, then $(\frac{\partial w}{\partial x})_{y,z} =$ _____.

2.2 Use Taylor's formula for $f(x, y)$ at the origin to find the quadratic approximation of $f(x, y) = \sin(x^2 + y^2)$ near the origin. _____.

2.3 Let $r = \sin 2\theta$, the equation of the tangent at $(1, \frac{\pi}{4})$ **in polar form** is _____.

2.4 The length of the curve $r = \sqrt{1 + \cos 2\theta}$, $0 \leq \theta \leq \pi\sqrt{2}$ is _____.

2.5 Set up the integral for evaluating the area of the region inside $r^2 = 6 \cos 2\theta$ and outside $r = \sqrt{3}$ is _____. (Do not evaluate the integral.)

(背面還有)

2.6 For the integral $\int_0^{3/2} \int_0^{9-4x^2} 16x \, dy \, dx$, write an equivalent double integral with the order of integration reversed. _____ . **(Do not evaluate the integral.)**

2.7 Set up the integral with the order $dz \, dr \, d\theta$ for evaluating the triple integral of the function $F(x, y, z) = \cos(z^3)$ over the solid region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 1$. _____ . **(Do not evaluate the integral.)**

2.8 Set up the integral with the order $d\rho \, d\phi \, d\theta$ for evaluating the volume of the solid bounded below by the sphere $\rho = 2 \cos \phi$ and above by the cone $z = \sqrt{x^2 + y^2}$. _____ . **(Do not evaluate the integral.)**

計算題

3. (12 points) Find the absolute extreme values of $f(x, y) = y^3 - 3xy + x^3$ on the set $D = \{(x, y) | x \geq 0, y \geq 0, x + y \leq 2\}$.

4. (12 points) Use the technique of Lagrange multipliers to find the points on the sphere $x^2 + y^2 + z^2 = 25$ where $f(x, y, z) = x + 2y + 3z$ has its maximum and minimum values. Also find those maximum and minimum values of $f(x, y, z)$.

5. (12 points) Evaluate $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \sin(x^2 + y^2) \, dx \, dy$.

6. (12 points) Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 \, dy \, dx$.