微積分聯合教學　段考一　**Oct 23, 2007**

考試時間 120 分鐘，試題總共 120 分。是非與填充題請在專用答案紙上作答，其他題目請寫在考試卷上。試題有兩張紙，共三面，滿分 120 分。答案紙與考試卷務必寫學號、姓名，試題不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試結束前 10 分鐘收回是非與填充題答案紙。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*18* points)，請答 **T** (True) 或 **F** (False)

1. If $f(x) = g(x)$ for all real numbers x except for $x = 0$, then

$$\lim_{x\to 0} f(x) = L \quad \Rightarrow \quad \lim_{x\to 0} g(x) = L$$

2. Since $\sec 0 = 1$ and $\sec \pi = -1$, so by the *intermediate value theorem* there is a number $x_0 \in (0,\, \pi)$ such that $\sec x_0 = 0$.

3. If $\lim\limits_{x\to a} f(x) = 0$, there must be a number c such that $|f(c)| < 10^{-5}$.

4. If $f(x)$ is differentiable at $x = a$, then $\big(f(x)\big)^3 + 5f(x)$ is differentiable at $x = a$.

5. $\lceil -x \rceil = -\lfloor x \rfloor$ for all real numbers x.

6. If the product function $h(x) = f(x)\cdot g(x)$ is continuous at $x = 0$, then $f(x)$ and $g(x)$ must be continuous at $x = 0$.

填充題 (*42* points)，其中 A–D 每格 3 分、E–J 每格 5 分

1. The volume $V = \dfrac{4}{3}\pi r^3$ of a spherical balloon changes with the radius r.

a. At what rate does the volume change with respect to the radius when $r = 2$ ft? A

b. Use the approximation $\Delta V \approx V'(r)\,\Delta r$ to estimate the increase of the volume when the radius changes from 2 to 2.2 ft. B

2. At time $t \geq 0$, the velocity of a body moving along the s-axis is $v = t^2 - 4t + 3$.

a. When is the body moving forward? C

b. When is the body's velocity increasing? D

(背面還有)

3. Let $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^3$, and $j(x) = 2x$. Express $u = \sqrt{x^3 - 3}$ as a composite function involving one or more of f, g, h and j. E

4. If $\lim_{x \to -2} \dfrac{f(x)}{x^2} = 1$, then

$$\lim_{x \to -2} \left(f(x) + \frac{f(x)}{x} \right) = \boxed{\text{F}}$$

5. Find the linearization $L(x)$ of $f(x) = \sqrt[3]{x}$ at $x = -8$. G

6. Find the derivative function for $y = \sin x \cos x$. H

7. Find the slope of the tangent line to the ellipse

$$x = a\cos t, \quad y = b\sin t, \qquad 0 \le t \le 2\pi$$

when $t = \pi/4$. I

8. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$. J

以下爲計算或問答題，請在考試卷上盡量依序作答，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Sketch the regions defined by the following inequalities. (在坐標平面上畫出滿足以下不等式的區域。)

a. $x^2 - 4x + y^2 \le 0$, **b.** $|x| + |y| \le 1$.

2. (*10* points) Find numbers A and B such that the function

$$f(x) = \begin{cases} x^3, & \text{if } x \le 1 \\ Ax + B, & \text{if } x > 1 \end{cases}$$

is differentiable at $x = 1$.

3. (*10* points) **a.** Use the fact $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ to find the value of

$$\lim_{x \to 0} \frac{3x}{\tan 5x}$$

b. Find the tangent line of the equation $\tan(xy) = x$ at the point $(1, \dfrac{\pi}{4})$.

4. (*10* points) Coffee is draining from a conical filter into a cylindrical coffee pot at the rate of 10 in^3/min. The top diameter of the cone, the height of the cone and the diameter of the cylinder are all 6 inches. When the coffee in the cone is 5 inches

deep, answer the following questions.

a. How fast is the level in the pot rising?

b. How fast is the level in the cone falling?

字彙：

coffee 咖啡

drain 滴漏

conical filter 圓錐狀的濾斗

cylindrical pot 圓柱形的壺

diameter 直徑

level 水平面 (這裡指咖啡的液面)

cone 圓錐

rise 上升

fall 下降

inch 英吋 (縮寫成 in)

min 分鐘 (minute 的縮寫)

(背面還有)

5. (*20* points) Let

$$\ell(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{for } x = 0. \end{cases}$$

a. For $x \neq 0$, find $\ell'(x)$.

b. Write down the definition of $\ell'(0)$ in the form of a limit of functions.

c. Show that $\ell'(0) = 0$. That is, we conclude that $\ell(x)$ is a differentiable function for all real numbers x.

d. Is the derivative function $\ell'(x)$ continuous at $x = 0$? Give reasons for your answer.

e. 一般而言，如果一個函數 $f(x)$ 可微，則它的導函數 $f'(x)$ 必然連續嗎？請說明理由。