微積分聯合教學　段考二　　Dec 4, 2007

考試時間 120 分鐘，試題有兩張紙，共三面，滿分 120 分。所有題目都請在考試卷上作答，而是非與填充題務必寫在第一頁。考試卷務必寫學號、姓名，試題不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*15* points)，請答 **T** (True) 或 **F** (False)

1. $\displaystyle\lim_{h\to 0}\frac{1}{h}\int_x^{x+h}\frac{1}{t^2+1}\,dt=\frac{1}{x^2+1}$

2. Let $A=\displaystyle\int_0^2\sqrt{1+x^3}\,dx$, then $0\le A\le 2$.

3. Let $f(x)$ be a continuous function on $[0,1]$, a is a constant and

$$F(x)=a+\int_0^x f(t)\,dt$$

for $x\in[0,1]$. Then $F(x)$ is the solution of the equation $y'=f(x)$ with the condition $y(0)=a$.

4. $\displaystyle\lim_{x\to\infty}x\sin\frac{1}{x}=\big(\lim_{x\to\infty}x\big)\cdot\sin\big(\lim_{x\to\infty}\frac{1}{x}\big)=\big(\lim_{x\to\infty}x\big)\cdot\sin 0=0\cdot\lim_{x\to\infty}x=0$

5. The polynomial $y=x^9+x-1$ has exactly one real root.

填充題 (*55* points)，[A]-[K] 每格 5 分

1. Find a value of m [A] that makes the function

$$f(x)=\begin{cases}3, & x=0\\ -x^2+5x+a, & 0<x<1\\ mx+b, & 1\le x\le 2\end{cases}$$

satisfies the hypotheses of the Mean Value Theorem on the interval $[0,2]$.

2. Find a value of c [B] that makes the function

$$f(x)=\begin{cases}\dfrac{3x-\sin 3x}{5x^3}, & x\neq 0\\ c, & x=0\end{cases}$$

continuous at $x=0$.

3. Find a curve $y = f(x)$ with the following properties: $\boxed{\text{C}}$

(i) $y'' = 8x$.

(ii) Its graph passes through the point $(0, 1)$.

(iii) It has a horizontal tangent line at $(0, 1)$.

4. Find the linearization of

$$g(x) = 5 + \int_1^{x^2} \sec(t-1)\,dt$$

at $x = -1$. $\boxed{\text{D}}$

5. Find the area $\boxed{\text{E}}$ of the region enclosed by $y = 4\sin x$ and $y = \sin 4x$, for $0 \le x \le \pi$.

6. Suppose that $F(x)$ is an antiderivative of $f(x) = \dfrac{\sin x}{x}$ for $x > 0$. Express

$$\int_1^3 \frac{\sin 3x}{x}\,dx$$

in terms of F. $\boxed{\text{F}}$

7. Find the length $\boxed{\text{G}}$ of the curve

$$y = \int_{-2}^{x} \sqrt{4t^4 - 1}\,dt$$

for $-2 \le x \le -1$.

8. Let V be the volume of the solid generated by revolving the region in the first quadrant bounded on the left by the circle $x^2 + y^2 = 3$, on the right by the line $x = \sqrt{3}$ and above by the line $y = \sqrt{3}$, about the y-axis. First express V by the washer method: $V = \displaystyle\int_0^{\sqrt{3}} \boxed{\text{H}}\,dy$, then express it by the shell method $V = \displaystyle\int_0^{\sqrt{3}} \boxed{\text{I}}\,dx$. 不必算出定積分。

9. Express the following limit

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1} + \frac{n}{n^2+4} + \cdots + \frac{n}{n^2+n^2}\right)$$

by a definite integral $\displaystyle\int_0^1 \boxed{\text{J}}\,dx$. 不必算出定積分。

10. Find the volume of the solid generated by revolving the triangle with vertices $(0,0)$, $(2,1)$ and $(1,3)$ about the x-axis. K

以下為計算或問答題，請在考試卷上盡量依序作答，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Sketch the graph of the polynomial function $y = x^4 + 2x^3$ with the coordinates of all roots, local extreme points and inflection points.

2. (*10* points) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 5.

3. (*10* points) 有一只截斷的圓錐形杯子，其縱剖面如圖示：底部直徑為 2 in (英吋)，頂部直徑為 3 in，高度為 4 in。假設杯內裝滿密度為 $\frac{4}{9}$ oz/in^3 的奶昔，吸管的高度為 8 in，求吸完一整杯奶昔所做的功。

4. (*10* points) (a) Let $f(x)$ be a continuous function on $[a,b]$ with $a < b$. Prove that if $\int_a^b f(x)\,dx = 0$ there must be at least one $c \in [a,b]$ such that $f(c) = 0$. (*4 points*)

(b) 請舉例說明：如果 $f(x)$ 不是連續函數，則結論可能不正確。 (*3 points*)

(c) 請舉例說明：如果 $a = b$，則結論可能不正確。 (*3 points*)

5. (*10* points) Let $f(x)$ be twice differentiable on $(0,1)$. Prove that if $f''(x) \neq 0$ for all $x \in (0,1)$ then f has at most two zeros in $(0,1)$.