

考試時間 120 分鐘，試題有兩張紙，共三面，滿分 120 分。所有題目都請在考試卷上作答，而非與填充題務必寫在第一頁。考試卷務必寫學號、姓名，試題不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (15 points)，請答 **T** (True) 或 **F** (False)

1. The function

$$f(x) = \begin{cases} \frac{3x - \sin 3x}{5x^3}, & x \neq 0 \\ \frac{3}{5}, & x = 0 \end{cases}$$

is continuous at $x = 0$.

2. The function $f(x) = x - \ln x$ is increasing.

3. If $a > b$, the function

$$f(x) = \frac{ax + b}{x + 1}$$

is one-to-one.

4. The function e^{x-1} grows slower than e^x as $x \rightarrow \infty$.

5. $\int_{-2}^2 \frac{1}{x} dx = \ln |x| \Big|_{-2}^2 = \ln 2 - \ln 2 = 0$.

填充題 (45 points)，- 每格 5 分

1. If $x^3 + y^3 = 9$, find the value of d^2y/dx^2 at the point $(1, 2)$.

2. Let

$$f(x) = \begin{cases} x^3, & \text{if } x \leq 1 \\ Ax + B, & \text{if } x > 1 \end{cases}$$

be differentiable at $x = 1$. Find $f(2)$.

(背面還有)

3. Let

$$g(x) = 5 + \int_1^{x^2} \sec(t - 1) dt.$$

Find $g(-1) + g'(-1)$. C

4. Find the derivative of y with respect to x D

$$y = x^{\sin x}$$

5. Evaluate the indefinite integral E

$$\int \frac{1}{1 + e^x} dx$$

6. Evaluate the indefinite integral F

$$\int \frac{1}{y^2 + 6y + 10} dy$$

7. Evaluate the indefinite integral G

$$\int \frac{1}{x - \sqrt{x}} dx$$

8. Evaluate the definite integral H

$$\int_0^\pi x^2 \sin x dx$$

9. For what values of p does the improper integral

$$\int_a^\infty \frac{1}{x^p} dx$$

converge for any number $a > 0$. I

以下為計算或問答題，請在考試卷上盡量依序作答，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) Show that the polynomial equation $y = x^9 + x - 1$ has exactly one real root.

2. (10 points) Evaluate the definite integral

$$\int_0^{\pi/6} 2 \cos^5 2x \, dx$$

3. (10 points) Evaluate the indefinite integral for $y > 3$

$$\int \frac{\sqrt{y^2 - 9}}{y^3} \, dy$$

4. (10 points) Use a test to determine if the following improper integral converge.

$$\int_2^{\infty} \frac{x}{\sqrt{x^4 - 1}} \, dx$$

If more than one method applies, use whatever method you prefer. 判斷收斂或發散即可，不必算出積分值。

5. (20 points) Let

$$T(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

(a) For what value of x does $T(x) = 0$?

(b) Find A such that

$$\frac{d}{dx} T(x) = \left(\frac{A}{e^x + e^{-x}} \right)^2$$

(c) Show that $T(x)$ has an inverse function $T^{-1}(x)$.

(d) Show that

$$\frac{d}{dx} T^{-1}(x) = \frac{1}{1 - x^2} \quad \text{for } |x| < 1.$$

(e) Show that

$$T^{-1}(x) = \frac{1}{2} \ln \frac{1+x}{1-x} \quad \text{for } |x| < 1.$$

[提示：即使你不會證明 (d) 還是可以引用那個事實。]