是非题 (15 points), 请答 T (True) 或 F (False)

1. Let \( u, v \) and \( w \) be space vectors, then \( (u \times v) \times w = u \times (v \times w) \).

2. The plane curve \( r(t) = (\ln \sec t)i + tj \) is smooth for \( -\pi/2 < t < \pi/2 \).

3. The velocity and acceleration vectors \( v \) and \( a \) are always orthogonal to each other for the helix \( r = \sin t i + tj + \cos t k \) for \( t \geq 0 \).

4. If the partial derivatives \( f_x \) and \( f_y \) of a function \( f(x, y) \) are continuous throughout an open region \( R \), then \( f(x, y) \) is continuous on \( R \).

5. \( \lim_{(x, y) \to (0, 0)} \frac{x^2}{x^2 + y^2} = 0. \)

填充题 (55 points), [A]-[K] 每格 5 分

1. Find the torsion \( \tau \) for the plane curve \( r(t) = (\ln \sec t)i + tj \) for \( -\pi/2 < t < \pi/2 \). [A]

2. Find the arc length of the plane curve \( r(t) = (\cos t + t \sin t)i + (\sin t - t \cos t)j \) for \( \pi/2 \leq t \leq \pi \). [B]

3. Let \( s \) be the arc length parameter of the plane curve \( r(t) = (\cos t + t \sin t)i + (\sin t - t \cos t)j \) for \( \pi/2 \leq t \leq \pi \). Find the derivative \( ds/dt \). [C]

4. Let \( u \) and \( v \) be nonzero space vectors. Use \( u \) and \( v \) to describe a vector that is always orthogonal to both \( u + v \) and \( u - v \). [D]

5. Find the distance from the point \((0, 0, 0)\) to the line \( x = 5 + 3t \), \( y = 5 + 4t \), \( z = -3 - 5t \). [E]

(背面還有)
6. The surface on the right is the graph of
\[ z = \frac{xy(x^2 - y^2)}{x^2 + y^2}. \]
Which of the following best represents the level curves of \( z \)? \( [F] \)

\[ \text{a.} \quad \text{b.} \quad \text{c.} \]

p.973, #14 \quad p.973, #15 \quad p.973, #18

7. Sketch the graph (named *dimpled limaçon*) of the equation \( r = \frac{3}{2} + \cos \theta \) in polar coordinates. \( [G] \)

8. How many points of intersection are there for the pair of curves \( r^2 = \sqrt{2} \sin \theta \) and \( r^2 = \sqrt{2} \cos \theta \) in polar coordinates? \( [H] \)

9. Find the value of \( \partial x / \partial z \) at the point \( (1, -1, -3) \) if the equation
\[ xz + y \ln x - x^2 + 4 = 0 \]
defines \( x \) as a function of the two independent variables \( y \) and \( z \) and the partial derivative exists. \( [I] \)

10. Find the smallest number \( \delta \) such that
\[ |(x, y) - (0, 0)| < \delta \Rightarrow |f(x, y) - f(0, 0)| < 0.01 \]
where \( f(x, y) = x^2 + y^2 \). \( [J] \)

11. Given the fact that
\[ 1 - \frac{x^2 y^2}{3} < \frac{\tan^{-1} xy}{xy} < 1, \]
find
\[ \lim_{(x,y) \to (0,0)} \frac{\tan^{-1} xy}{xy} = [K]. \]
以下为计算或问答题，请在考试卷上尽量依序作答，可以用中文或英文作答。请详细计算过程，否则不予计分。需标明题号但不必抄题。

1. (10 points) Find the unit tangent vector $T$, unit normal vector $N$ and curvature $\kappa$ for the plane curve $\mathbf{r}(t) = (\ln \sec t) \mathbf{i} + t \mathbf{j}$ for $-\pi/2 < t < \pi/2$.

2. (10 points) Find the linearization $L(x, y)$ of

$$f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$$

at the point $(3, 2)$. Let the approximation error be $E(x, y) = f(x, y) - L(x, y)$, find the smallest theoretical upper bound for $|E(x, y)|$ over the rectangle

$$R : |x - 3| \leq 0.1, \quad |y - 2| \leq 0.1.$$

3. (10 points) Among all the points on the graph of $z = 10 - x^2 - y^2$ that lie above the plane $x + 2y + 3z = 0$, find the point farthest from the plane. And find the distance from that point to the plane.

4. (10 points) Is there a direction $\mathbf{u}$ in which the rate of change of the temperature function $T(x, y, z) = 2xy - yz$ at $P(1, -1, 1)$ equals $-3$ (that is, $D_u T(1, -1, 1) = -3$)? If there is, find it. If there is not, give reasons for your answer.

5. (10 points) Let $T = g(x, y)$ be the temperature at the point $(x, y)$ on the ellipse

$$x = 2\sqrt{2}\cos t, \quad y = \sqrt{2}\sin t, \quad 0 \leq t \leq 2\pi.$$

Suppose that

$$\frac{\partial T}{\partial x} = y, \quad \frac{\partial T}{\partial y} = x.$$

Locate the maximum and minimum temperatures on the ellipse by examining $dT/dt$ and $d^2T/dt^2$. 

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