微積分聯合教學　段考四　　Apr 1, 2008

考試時間 120 分鐘，試題有兩張紙，共三面，滿分 120 分。所有題目都請在考試卷上作答，而是非與填充題必須寫在第一頁。考試卷務必寫學號、姓名，試題不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*15* points)，請答 **T** (True) 或 **F** (False)

1. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be space vectors, then $(\mathbf{u}\times\mathbf{v})\times\mathbf{w} = \mathbf{u}\times(\mathbf{v}\times\mathbf{w})$.

2. The plane curve $\mathbf{r}(t) = (\ln\sec t)\mathbf{i} + t\,\mathbf{j}$ is smooth for $-\pi/2 < t < \pi/2$.

3. The velocity and acceleration vectors $\mathbf{v}$ and $\mathbf{a}$ are always orthogonal to each other for the helix $\mathbf{r} = \sin t\,\mathbf{i} + t\,\mathbf{j} + \cos t\,\mathbf{k}$ for $t \geq 0$.

4. If the partial derivatives f_x and f_y of a function $f(x, y)$ are continuous throughout an open region R, then $f(x, y)$ is continuous on R.

5. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} = 0.$

填充題 (*55* points)，[A]–[K] 每格 5 分

1. Find the torsion τ for the plane curve $\mathbf{r}(t) = (\ln\sec t)\mathbf{i} + t\,\mathbf{j}$ for $-\pi/2 < t < \pi/2$. [A]

2. Find the arc length of the plane curve $\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}$ for $\pi/2 \leq t \leq \pi$. [B]

3. Let s be the arc length parameter of the plane curve $\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}$ for $\pi/2 \leq t \leq \pi$. Find the derivative ds/dt. [C]

4. Let $\mathbf{u}$ and $\mathbf{v}$ be nonzero space vectors. Use $\mathbf{u}$ and $\mathbf{v}$ to describe a vector that is always orthogonal to both $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$. [D]

5. Find the distance from the point $(0, 0, 0)$ to the line $x = 5 + 3t$, $y = 5 + 4t$, $z = -3 - 5t$. [E]

(背面還有)

6. The surface on the right is the graph of

$$z = \frac{xy(x^2 - y^2)}{x^2 + y^2}.$$

Which of the following best represents the level curves of z? $\boxed{\text{F}}$

a. p.973, #14

b. p.973, #15

c. p.973, #18

7. Sketch the graph (named *dimpled limaçon*) of the equation $r = \frac{3}{2} + \cos\theta$ in polar coordinates. $\boxed{\text{G}}$

8. How many points of intersection are there for the pair of curves $r^2 = \sqrt{2}\sin\theta$ and $r^2 = \sqrt{2}\cos\theta$ in polar coordinates? $\boxed{\text{H}}$

9. Find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ if the equation

$$xz + y\ln x - x^2 + 4 = 0$$

defines x as a function of the two independent variables y and z and the partial derivative exists. $\boxed{\text{I}}$

10. Find the smallest number δ such that

$$|(x, y) - (0, 0)| < \delta \Longrightarrow |f(x, y) - f(0, 0)| < 0.01$$

where $f(x, y) = x^2 + y^2$. $\boxed{\text{J}}$

11. Given the fact that

$$1 - \frac{x^2y^2}{3} < \frac{\tan^{-1} xy}{xy} < 1,$$

find

$$\lim_{(x,y)\to(0,0)} \frac{\tan^{-1} xy}{xy} = \boxed{\text{K}}.$$

以下爲計算或問答題，請在考試卷上盡量依序作答，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the unit tangent vector $\mathbf{T}$, unit normal vector $\mathbf{N}$ and curvature κ for the plane curve $\mathbf{r}(t) = (\ln \sec t)\mathbf{i} + t\,\mathbf{j}$ for $-\pi/2 < t < \pi/2$.

2. (*10* points) Find the linearization $L(x, y)$ of

$$f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$$

at the point $(3, 2)$. Let the approximation error be $E(x, y) = f(x, y) - L(x, y)$, find the smallest theoretical upper bound for $|E(x, y)|$ over the rectangle

$$R: \quad |x - 3| \leq 0.1, \quad |y - 2| \leq 0.1.$$

3. (*10* points) Among all the points on the graph of $z = 10 - x^2 - y^2$ that lie above the plane $x + 2y + 3z = 0$, find the point farthest from the plane. And find the distance from that point to the plane.

4. (*10* points) Is there a direction $\mathbf{u}$ in which the rate of change of the temperature function $T(x, y, z) = 2xy - yz$ at P(1,-1,1) equals -3 (that is, $D_{u}T(1, -1, 1) = -3$)? If there is, find it. If there is not, give reasons for your answer.

5. (*10* points) Let $T = g(x, y)$ be the temperature at the point (x, y) on the ellipse

$$x = 2\sqrt{2}\cos t, \quad y = \sqrt{2}\sin t, \quad 0 \leq t \leq 2\pi.$$

Suppose that

$$\frac{\partial T}{\partial x} = y, \quad \frac{\partial T}{\partial y} = x.$$

Locate the maximum and minimum temperatures on the ellipse by examining dT/dt and d^2T/dt^2.