微積分聯合教學　段考五　May 6, 2008

考試時間 120 分鐘，試題有兩張紙，共四面，滿分 120 分。所有題目都請在考試卷上作答，而是非與填充題必須寫在第一頁。考試卷務必寫學號、姓名，試題不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*15* points)，請答 **T** (True) 或 **F** (False)

1. The maximum value of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ happens at the point (x,y) where $x\mathbf{i} + y\mathbf{j}$ is parallel to $3\mathbf{i} + 4\mathbf{j}$.

2. If $\int_0^1 \int_0^1 f(x,y)\,dx\,dy = 0$ and $\int_0^1 \int_0^1 f(x,y)\,dy\,dx = 1$, then $f(x,y)$ must *not* be continuous in the square $[0,1] \times [0,1]$.

3. The double integral of

$$\frac{1}{1 - x^2 - y^2}$$

over the unit disk $x^2 + y^2 \le 1$ exists.

4. The value of the double integral

$$\int_{\pi/4}^{3\pi/4} \int_{\csc\theta}^{2\sin\theta} r\,dr\,d\theta$$

is the area of the shaded region in the accompanying figure.

5. The value of the triple integral

$$\int_0^1 \int_0^{2-2x} \int_0^{6-6x-3y} dz\,dy\,dx$$

is the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane passing through $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$.

(背面還有)

填充題 (55 points)，A–K 每格 5 分

1. Find the area of the triangle bounded by the following lines. A

$$y = x, \quad y = 2x \quad \text{and} \quad x + y = 2$$

2. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \le x \le 2\ln 2$ and $\ln 2 \le y \le 2\ln 2$. B

3. Find the length of the curve $r = a\sin^2(\theta/2)$ for $0 \le \theta \le \pi$ and $a > 0$. C

4. Evaluate the integral

$$\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{y^2 z}\, dy\, dx\, dz$$

by changing the order of integration in an appropriate way. D

5. Convert the integral

$$\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^x x^2 + y^2\, dz\, dx\, dy$$

to an equivalent integral in cylindrical coordinates. E (只要寫出積分式，不要計算積分值)

6. Find the spherical coordinate limits for the integral that calculates the volume of the portion of the sphere $\rho \le 2$ between the cone $\phi = \pi/3$ and the xy-plane. F (只要寫出積分式，不要計算積分值)

7. A fluid's velocity field is $\mathbf{F} = x\mathbf{i} + z\mathbf{j} + y\mathbf{k}$. Find its flow along the helix $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$ for $0 \le t \le \pi/2$. G

8. Evaluate

$$\int_C (y^2 + 2x + 1)\, dx + (2xy + 4y - 1)\, dy$$

along the path C: $y = x^2$ from $(0, 0)$ to $(1, 1)$. H

(背面也有)

9. Change the double integral

$$\iint_R 3x^2 + 14xy + 8y^2 \, dx \, dy$$

to an equivalent integral in the uv-plane with $u = 3x+2y$ and $v = x+4y$, where R is the parallelogram bounded by lines $3x + 2y = 2$, $3x + 2y = 6$, $x + 4y = 0$ and $x + 4y = 4$. [I] (只要寫出積分式，不要計算積分值)

10. Evaluate the following integral [J]

$$\int_0^\infty \int_0^\infty \frac{1}{(1 + x^2 + y^2)^2} \, dx \, dy$$

11. Find the flow of the field $\mathbf{F} = \nabla(xy^2z^3)$ along the line segment from $(1, 1, 1)$ to $(2, 1, -1)$. [K]

以下為計算或問答題，請在考試卷上盡量依序作答，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Sketch the region of the integration for

$$\int_0^2 \int_0^{4-x^2} \frac{x\, e^{2y}}{4 - y} \, dy \, dx$$

Then evaluate the integral.

2. (*10* points) Change the Cartesian integral

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4\sqrt{x^2 + y^2}}{1 + x^2 + y^2} \, dx \, dy$$

to an equivalent polar integral. Then evaluate the polar integral.

3. (*10* points) Evaluate

$$\iiint |xyz| \, dx \, dy \, dz$$

over the solid ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$$

by changing variables $x = au$, $y = bv$ and $z = cw$.

(背面還有)

4. (*10* points) Find the volume of the region common to the cylinders $x^2+y^2=1$ and $x^2+z^2=1$. One-eighth of which is shown in the accompanying figure.

5. (*10* points) Show that the maximum value of $x^2y^2z^2$ on the sphere $x^2+y^2+z^2=r^2$ is

$$\left(\frac{r^2}{3}\right)^3$$

Then prove the three-term inequality for geometric and arithmetic means:

$$\sqrt[3]{abc} \leq \frac{a+b+c}{3}, \quad \text{where } a \geq 0,\ b \geq 0,\ \text{and } c \geq 0.$$

(背面也有)