

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷上)

1. If $f(x)$ is continuous at $x = c$, then $f(x)$ is differentiable at $x = c$.

2. There is a value of b that will make

$$g(x) = \begin{cases} x + b, & x < 0 \\ \sin x, & x \geq 0 \end{cases}$$

differentiable at $x = 0$.

3. If f is an odd function, then $\lim_{x \rightarrow 0} f(x) = 0$.

4. $\lim_{x \rightarrow \infty} [f(x) + g(x)] = \lim_{x \rightarrow \infty} f(x) + \lim_{x \rightarrow \infty} g(x)$ is true.

5. $f(x) = \left| \frac{x \sin x}{x^2 + 2} \right|$ is a continuous function for all $x \in \mathbb{R}$.

6. The domain of $f(x) = \sqrt{\frac{x}{1-x}}$ is $[0, 1)$.

7. If the product function $f \cdot g$ is continuous at $x = 0$, then $f(x)$ and $g(x)$ are continuous at $x = 0$.

8. If $f(x) = \sqrt{x}$, then $f(x)$ is differentiable at $x = 0$.

9. Suppose $f(x)$ is continuous on $[a, b]$ and $f(a) \leq f(b) < m$. Then there must exist $c \in [a, b]$ such that $f(c) = m$.

10. The graph of

$$g(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

has a tangent line at the origin $(0, 0)$.

(下頁還有試題)

填充題 (40 points), 每題 5 分。(請依題號順序依序寫在答案卷上)

1. Find $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$. Answer : = _____.
2. If $f'(0) = -1$, find $\lim_{h \rightarrow 0} \frac{f(3h) - f(-2h)}{h}$. Answer : _____.
3. Find two asymptotes of the graph of $f(x) = \frac{x^2 + 3x - 17}{3x - 9}$.
Answer : _____, _____.
4. At time $t \geq 0$, the position of a body moving along the s -axis is $s = t^3 - 6t^2 + 9t$.
Find the time interval on which the body is moving backward.
Answer : _____.
5. A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x -coordinate (measured in meters) increases at a steady 10m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m? Answer : _____.
6. Let $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x}$, find the domain of the composite function $g \circ f$. Answer : _____.
7. Find the tangent and normal lines of the curve $x \sin 2y = y \cos 2x$ at the point $(\frac{\pi}{4}, \frac{\pi}{2})$. Answer : (Tangent line) _____, (Normal line) _____.
8. Find $\frac{d^{20}}{dx^{20}} [(\cos \frac{x}{2})(\sin \frac{x}{2})]$. Answer : = _____.

計算問答證明題 (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Assume $f(x) = \sqrt{1-5x}$, $x_0 = -3$, $\epsilon = 0.5$, find $L = \lim_{x \rightarrow x_0} f(x)$ and then find a number $\delta > 0$ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$.
2. (10 points) Explain why the equation $\cos x = x$ has at least one solution.
3. (10 points) Give an $\epsilon - \delta$ proof for the limit $\lim_{x \rightarrow \sqrt{6}} \frac{1}{x^2} = \frac{1}{6}$.

(下頁還有試題)

4. (10 points) Find dy/dx of the curve $x^2 \cos^2 y = \sin y$ by using implicit differentiation and find the line that is tangent to the curve at the point $(0, \pi)$.

5. (10 points) Assume $x = \sec^2 t - 1, y = 2 \tan t$.

Find an equation for the line tangent to the curve at the point where $t = -\frac{\pi}{4}$.

Also, find the value of d^2y/dx^2 at this point.

6. (10 points) Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0. \\ 0, & x = 0. \end{cases}$$

a. Show that $f'(0) = 0$.

b. Is the function $f'(x)$ continuous at $x = 0$? Explain.