微積分聯合教學　段考二　Dec 2, 2008

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目都**請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁**。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷上)

1. The area of the region between the x-axis and the graph of $f(x) = -x^3+x^2+2x$ is $\displaystyle\int_{-1}^{2} -x^3+x^2+2x\ dx = \frac{27}{12}$.

2. f is a continuous function and a, b, c are arbitrary real numbers. Then $\displaystyle\int_b^a f(x)\ dx - \int_c^a f(x)\ dx = \int_b^c f(x)\ dx$.

3. $\displaystyle\int_{-1}^{1} \sqrt{x^2-x^4}\ dx = \int_{-1}^{1} x\sqrt{1-x^2}\ dx$.

4. If an odd function $g(x)$ has a local minimum value at $x=c$, then g assumes a local maximum at the point $x=-c$.

5. Let $a<b<c$ and f is a function defined on $[a,c]$. If f is increasing on $[a,b]$ and increasing on $(b,c]$, then f is increasing on $[a,c]$.

6. $\displaystyle\int_{-2}^{4} |x+1|\ dx = 13$.

7. The graph of every polynomial of even degree has at least one horizontal tangent.

8. If $x=g(y)$ is continuously differentiable on $[c,d]$, the area of the surface generated by revolving the curve $x=g(y)$ about the y-axis is $S = \displaystyle\int_c^d 2\pi g(y)\sqrt{1+(g'(y))^2}\ dy$.

9. If $f(x)=x$, then the average value of f on $[0,2]$ is 1.

(下頁還有試題)

10. The integral of every polynomial of odd degree on the symmetric interval $[-a, a]$ is zero, where $a > 0$.

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷上)

1. Find $\lim_{x\to 0} \frac{1}{3x^2} \int_{x^2}^{0} \cos t \, dt$. Answer：___________.

2. Find a curve $y = f(x)$ with the following properties：

a. $y'' = 2 - 6x$.

b. Its graph passes through the point $(0, 1)$.

c. The slope of the tangent line to the curve at $(0, 1)$ is equal to 4.

Answer：___________.

3. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be? Answer：___________.

4. Evaluate $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta \sin^4 \sqrt{\theta}}} \, d\theta$. Answer：___________.

5. Let $F(x) = \int_0^x f(t) \, dt$ and f is an odd continuous function. If $F(1) = 4$, find $F(-1)$. Answer：___________.

6. Find the area of the surface generated by revolving the curve $x = \cos t$, $y = 2 + \sin t$, $0 \leq t \leq 2\pi$ about the x-axis. Answer：___________.

7. $f(x) = \begin{cases} x^2, & x \text{ is an integer} \\ \sin x \cos x, & x \text{ is not an integer.} \end{cases}$

Find $\int_0^5 f(x) \, dx$. Answer：___________.

8. Let V be the volume of the solid generated by revolving the region in the first quadrant that is bounded above by the curve $y = 1/\sqrt[3]{x}$, on the left by the line $x = 1/8$, and below by the line $y = 1$, about the line $x = -1$. Express V by the washer method. Write V in the form $\int_{\_\_}^{\_\_} \_\_\_\_\_\_\_ \, dy$. (不需算出其值)

Answer：___________.

(下頁還有試題)

計算問答證明題(*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。**請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the length of the curve $y = x^{\frac{2}{3}}$ from $x = 0$ to $x = 8$.

2. (*10* points) A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

3. (*10* points) Express the following limit as a definite integral and then evaluate the integral.

$$\lim_{n\to\infty}\left(\sum_{j=1}^{n}\frac{\sqrt{n^2-j^2}}{n^2}\right).$$

4. (*10* points) Find the limits,

a. $\lim\limits_{x\to 0}\dfrac{\sqrt{a(a+x)}-a}{x}, a > 0.$

b. $\lim\limits_{x\to 0}(\dfrac{1}{\sin x}-\dfrac{1}{x}).$

5. (*10* points) Let $f(x) = \frac{1}{4}x^4 - x^3$.

a. Find the intervals where f is increasing and where f is decreasing.

b. Find the local maxima and local minima of f.

c. Find the intervals where f is concave up and where f is concave down.

d. Find the inflection points of f.

e. Sketch f.

6. (*10* points)

a. A number a is called a **fixed point** of a function f if $f(a) = a$.

Prove that if $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point.

b. Show that for any numbers a and b, the inequality $|\cos b - \cos a| \leq |b - a|$ is true.