

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷上)

1. $\int_{-1}^1 \sqrt{x^2} dx = 0.$

2. We may use l'Hôpital's rule to get $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{e^x} = \lim_{x \rightarrow 0} 2e^x = 2.$

3. Let $g : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \mapsto \mathbb{R}$ be defined by

$$g(x) = \begin{cases} x + b, & x < 0 \\ \tan x, & x \geq 0. \end{cases}$$

There is a value of b such that $g(x)$ is differentiable at $x = 0$.

4. If a composite $f \circ g$ is one-to-one, g must be one-to-one.

5. If $f(x) = 2^x$, then $f'(x) = x2^{x-1}$.

6. We can use the trigonometric substitution $x = 2 \tan \theta$ to evaluate the definite integral as follow: $\int_0^2 \frac{1}{(x^2 + 4)^2} dx = \int_0^2 \frac{1}{8 \sec^2 \theta} d\theta.$

7. If the product function $f \cdot g$ is continuous at $x = 0$, then $f(x)$ and $g(x)$ are continuous at $x = 0$.

8. If $f(x) = \ln \pi$, then $f'(x) = \frac{1}{\pi}$.

9. The point $(r, \theta) = \left(\frac{1}{2}, \frac{3\pi}{2}\right)$ lies on the curve $r = -\sin\left(\frac{\theta}{3}\right).$

10. $\int_0^3 \frac{1}{x-1} dx = \ln|x-1| \Big|_0^3 = \ln 2 - \ln 1 = \ln 2.$

(下頁還有試題)

填充題 (40 points), 每題 5 分。(請依題號順序依序寫在答案卷上)

1. Let $f(t) = (\sqrt{t})^{\ln t}$. Find $f'(t)$. Answer : = _____.
2. Assume $f'(0) = -1$. Evaluate $\lim_{h \rightarrow 0} \frac{f(3h) - f(-4h)}{h}$. Answer : _____.
3. Let $\alpha = \csc^{-1} \left(\frac{-\sqrt{5}}{2} \right)$. Find $\tan \alpha$. Answer : _____.
4. Find the length of the curve $y = \ln(\sin x)$, $\frac{\pi}{3} \leq x \leq \frac{\pi}{2}$. Answer : _____.
5. Let V be the volume of the solid generated by revolving the region in the first quadrant that is bounded above by the curve $y = 1/\sqrt[3]{x}$, on the left by the line $x = 1/8$, and below by the line $y = 1$, about the line $x = -1$. Express V by the washer method. Write V in the form $\int_{-}^{-} \text{_____} dy$. (不需算出其值)
Answer : _____.
6. Evaluate $\int e^{-2x} \cos 2x dx$. Answer : _____.
7. Find $\lim_{n \rightarrow \infty} \frac{1}{n} (e^{\frac{1}{n}} + e^{\frac{2}{n}} + \dots + e^{\frac{n-1}{n}} + e^{\frac{n}{n}})$. Answer : _____.
8. Find the all values of p such that the improper integral $\int_{0.1}^{\infty} \frac{1}{x^p} dx$ converges.
Answer : = _____.

計算問答證明題(60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the following limits.

a. $\lim_{x \rightarrow \infty} (1 + 2x)^{1/(3 \ln x)} =$

b. $\lim_{x \rightarrow 0} (e^x + 3x)^{1/x} =$

2. (10 points) Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

3. (10 points) Evaluate the integral

$$\int \frac{e^{4t} + 16}{(e^{2t} + 4)^2} dt.$$

(下頁還有試題)

4. (10 points)

a. Let $y = \log_2 \left(\frac{2^x \ln x}{(\ln 2)\sqrt{x+1}} \right)$. Find $\frac{dy}{dx}$.

b. Let $f(x) = 2x^3 + x$. Evaluate df^{-1}/dx at $x = 255 = f(5)$.

5. (10 points) Evaluate the integral

$$\int \frac{t+9}{t^2+9} dt.$$

6. (10 points) Find the tangent line to the curve

$$x^2 \cos^2 y = \sin y$$

at the point $(0, \pi)$.