微積分聯合教學　段考四　　Mar 31, 2009

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目都**請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁**。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。(請依題號順序依序寫在答案卷上)

1. If $f(x,y)$ has partial derivatives $f_x(x_0,y_0)$ and $f_y(x_0,y_0)$ then it is differentiable at (x_0,y_0).

2. Any closed region in the plane must be bounded.

3. If a function $f(x,y)$ has the same limit along two certain different paths as (x,y) approaches (x_0,y_0), then $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ must exist.

4. $(\mathbf{i}-\mathbf{k})\times(\mathbf{j}+\mathbf{k}) = \mathbf{i}-\mathbf{j}+\mathbf{k}$.

5. Let θ denote the angle between two 3-dimensional nonzero vectors $\mathbf{u}$ and $\mathbf{v}$. If $0<\theta<\dfrac{\pi}{4}$, then $\mathbf{u}\cdot\mathbf{v} > |\mathbf{u}\times\mathbf{v}|$.

6. Given three points $P(1,-2,6), Q(0,6,4)$ and $R(-8,1,-5)$, then the distance from P to line $\overleftrightarrow{QR}$ is larger than the distance from Q to line $\overleftrightarrow{PR}$.

7. If $\mathbf{r}$ is a differentiable vector function of t of constant length, then $\mathbf{r}\cdot\dfrac{d\mathbf{r}}{dt}\neq 0$.

8. The region $R=\{(x,y)\big|1\le x^2+y^2<2\}$ is open in the xy-plane.

9. If $\mathbf{u}\neq\mathbf{0}$ and if $\mathbf{u}\times\mathbf{v}=\mathbf{u}\times\mathbf{w}$ and $\mathbf{u}\cdot\mathbf{v}=\mathbf{u}\cdot\mathbf{w}$, then $\mathbf{v}=\mathbf{w}$.

10. Let function $f(x,y)$ be differentiable in an open region containing $P_0(x_0,y_0)$. Then, at point P_0, f decreases most rapidly in the direction of gradient vector ∇f at P_0.

(下頁還有試題)

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷上)

1. The plane $y = 2$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$. Answer：=___________.

2. Find $\dfrac{\partial w}{\partial u}$ when $u = v = 0$, if $w = x^2 + \dfrac{y}{x}$ and $x = u - 2v + 1$, $y = 2u + v - 2$. Answer：___________.

3. Find the limit, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{2y}\sin x}{\sin 2x}$. Answer：___________.

4. If $\mathbf{u}(t) = (\cos t, \sin t, t)$ and $\mathbf{v}(t) = (\sin t, \cos t, t)$, find $\dfrac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)]$. Answer：___________.

5. Find the angle between the planes $2x - 3y - 6z = 12$ and $x - 2y - 2z = 7$. Answer：___________.

6. Find the length of the curve $\mathbf{r}(t) = (\sqrt{2}t)\mathbf{i} + (\sqrt{2}t)\mathbf{j} + (1 - t)\mathbf{k}$ from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$. Answer：___________.

7. Assuming the equation $2xy + e^{x+y} - 2 = 0$ define y as a differentiable function of x, find the value of dy/dx at point $P(0, \ln 2)$. Answer：___________.

8. If $\mathbf{r}(t) = (t - \sin t, 1 - \cos t)$, find the maximum value of $|\mathbf{v}(t)|$. Answer：=___________.

計算問答證明題(*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。**請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find parametric equations for the line in which the planes $3x-6y-2z = 15$ and $2x + y - 2z = 5$ intersect.

2. (*10* points) Find $\mathbf{T}, \mathbf{N}$, and κ for the space curve $\mathbf{r}(t) = (6\sin 2t)\mathbf{i} + (6\cos 2t)\mathbf{j} + 5t\mathbf{k}$.

3. (*10* points) The derivative of $f(x, y)$ at $P_0(1, 2)$ in the direction of $\mathbf{i} + \mathbf{j}$ is $2\sqrt{2}$ and in the direction of $-2\mathbf{j}$ is -3. Find the derivative of f at $P_0(1, 2)$ in the direction of $-\mathbf{i} - 2\mathbf{j}$.

(下頁還有試題)

4. (*10* points) Let $f(x,y) = \dfrac{3x^2y}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin? Explain your answer and show your reasons.

5. (*10* points) Find the equations of the tangent plane and normal line to the surface $z = 2x\cos y - ye^x$ at $(0,0,0)$.