1. The function $f(x, y) = x^4y^4$ has a local minimum at $(-5, 0)$.

2. The image of $\mathbb{R}^2$ under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(\theta, z) = (\cos \theta, \sin \theta, z)$ is a cylinder.

3. If a function $f(x, y)$ is defined and continuous on the $xy$-plane, then $f(x, y)$ has absolute maxima.

4. The area of the fan-shaped region between the origin and the curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$, is $\int_{\alpha}^{\beta} \int_{0}^{f(\theta)} r \, dr \, d\theta$.

5. The area of the region that lies inside the cardioid curve $r = \cos \theta + 1$ and outside the circle $r = 1$ is $\frac{1}{2} \int_{0}^{2\pi} [(\cos \theta + 1)^2 - 1] \, d\theta$.

6. The function $f(x, y) = xy$ defined on the square bounded by the lines $x = 1$, $x = -1$, $y = 1$, and $y = -1$ has no absolute extreme value.

Fill in the blanks (40 points), each 5 points. (Please write the answers in the answer box)

1. Find all the points on the curve $x^2y = 2$ nearest the origin.
   Answer: __________.

2. Evaluate $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{0} \frac{4}{1 + x^2 + y^2} \, dx \, dy$. Answer: __________.

(Continued on the next page)
3. Consider \( \begin{cases} u = x + 2y \\ v = x - y. \end{cases} \)
Find the value of Jacobian \( \frac{\partial (x, y)}{\partial (u, v)}. \) Answer: 

4. Find the area of the region inside the cardioid \( r = 1 + \sin \theta \) and outside the circle \( r = \sin \theta. \) Answer: 

5. Write the spherical coordinate equation for \( z = \sqrt{3(x^2 + y^2)}. \) Answer: 

6. The integral \( \int_0^1 \int_x^1 F(x, y, z) \, dz \, dy \, dx \) is the triple integral of a function \( F(x, y, z) \) over the tetrahedron \( D \) with vertices \((0, 0, 0), (1, 1, 0), (0, 1, 0) \) and \((0, 1, 1). \) Rewrite the integral as an equivalent iterated integral in the order \( dy \, dz \, dx. \) Answer: 

7. Convert the integral \( \int_{-2}^{2} \int_{\sqrt{4-x^2}}^{\sqrt{8-x^2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \, dz \, dy \, dx \) to an equivalent integral in spherical coordinates. (Do not evaluate the integral.) Answer: 

8. Let \( f(x, y) \) be a continuous function. Write \( \int_0^8 \int_{x^2}^{32 \sqrt{x}} f(x, y) \, dy \, dx \) as an iterated integral in the order \( dx \, dy. \) Answer: 

計算問題(62 points)，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) Let \( f(x, y) = xy^2 - 2xy + 2x^2 - 15x. \) Find the local maxima, local minima and saddle points of \( f. \)

2. (10 points) Evaluate \( \int_R \frac{x - 2y}{3x - y} \, dA, \) where \( R \) is the parallelogram enclosed by the lines \( x - 2y = 0, x - 2y = 4, 3x - y = 1, \) and \( 3x - y = 8. \)

3. (10 points)
   a. Sketch \( r = 1 + \cos \theta \) in polar coordinates.
   b. Find the length of the graph \( r = 1 + \cos \theta \) in polar coordinates.
   c. Find the area of the region enclosed by \( r = 1 + \cos \theta \) in polar coordinates.
4. (12 points) Evaluate the following integrals.
   a. $\int_0^1 \int_y^1 \frac{\sin x}{x} \, dx \, dy$.
   b. $\int_0^8 \int_{\frac{1}{\sqrt{x}}}^2 \frac{1}{y^4 + 1} \, dy \, dx$.

5. (10 points) Find the limits (上・下限) of integration in cylindrical coordinates for integrating a function $f$ over the region $D$ bounded below by the plane $z = 0$, laterally by the circular cylinder $x^2 + (y - 1)^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.

6. (10 points) Find the absolute extrema of $f(x, y) = x^2 + y^2 + 3xy + 2$ on the set $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$.