微積分聯合教學　段考五　May 5, 2009

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目都**請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁**。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*18* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。(請依題號順序依序寫在答案卷上)

1. The function $f(x,y) = x^4y^4$ has a local minimum at $(-5,0)$.

2. The image of $\mathbb{R}^2$ under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(\theta, z) = (\cos\theta, \sin\theta, z)$ is a cylinder.

3. If a function $f(x,y)$ is defined and continuous on the xy-plane, then $f(x,y)$ has absolute maxima.

4. The area of the fan-shaped region between the origin and the curve $r = f(\theta)$, $\alpha \le \theta \le \beta$, is $\displaystyle\int_\alpha^\beta \int_0^{f(\theta)} r\,dr\,d\theta$.

5. The area of the region that lies inside the cardioid curve $r = \cos\theta + 1$ and outside the circle $r = 1$ is $\displaystyle\frac{1}{2}\int_0^{2\pi} [(\cos\theta+1)^2 - 1]\,d\theta$.

6. The function $f(x,y) = xy$ defined on the square bounded by the lines $x = 1$, $x = -1$, $y = 1$, and $y = -1$ has no absolute extreme value.

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷上)

1. Find all the points on the curve $x^2y = 2$ nearest the origin.

Answer：____________.

2. Evaluate $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4}{1+x^2+y^2}\,dx\,dy$.　Answer：____________.

(下頁還有試題)

3. Consider $\begin{cases} u = x + 2y \\ v = x - y. \end{cases}$
Find the value of Jacobian $\partial(x, y)/\partial(u, v)$. Answer： ___________.

4. Find the area of the region inside the cardioid $r = 1 + \sin\theta$ and outside the circle $r = \sin\theta$. Answer： ___________.

5. Write the spherical coordinate equation for $z = \sqrt{3(x^2 + y^2)}$.
Answer： ___________.

6. The integral $\displaystyle\int_0^1 \int_x^1 \int_0^{y-x} F(x, y, z)\, dz\, dy\, dx$ is the triple integral of a function $F(x, y, z)$ over the tetrahedron D with vertices $(0, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$ and $(0, 1, 1)$. Rewrite the integral as an equivalent iterated integral in the order $dy\, dz\, dx$. Answer： ___________.

7. Convert the integral $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} dz\, dy\, dx$ to an equivalent integral in spherical coordinates. (Do not evaluate the integral.) Answer： ___________.

8. Let $f(x, y)$ be a continuous function. Write $\displaystyle\int_0^8 \int_{x^2}^{32\sqrt[3]{x}} f(x, y)\, dy\, dx$ as an iterated integral in the order $dx\, dy$. Answer： ___________.

計算問答證明題(*62* points)，請依題號順序依序寫在答案卷上，可以用中文或英文作答。**請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Let $f(x, y) = xy^2 - 2xy + 2x^2 - 15x$. Find the local maxima, local minima and saddle points of f.

2. (*10* points) Evaluate $\displaystyle\iint_R \frac{x - 2y}{3x - y}\, dA$, where R is the parallelogram enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, and $3x - y = 8$.

3. (*10* points)

a. Sketch $r = 1 + \cos\theta$ in polar coordinates.

b. Find the length of the graph $r = 1 + \cos\theta$ in polar coordinates.

c. Find the area of the region enclosed by $r = 1 + \cos\theta$ in polar coordinates.

(下頁還有試題)

4. (*12* points) Evaluate the following integrals.

a. $\displaystyle\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy.$

b. $\displaystyle\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx.$

5. (*10* points) Find the limits (上、下限) of integration in cylindrical coordinates for integrating a function f over the region D bounded below by the plane $z = 0$, laterally by the circular cylinder $x^2 + (y-1)^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.

6. (*10* points) Find the absolute extrema of $f(x, y) = x^2 + y^2 + 3xy + 2$ on the set $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$.