微積分聯合教學　期末考　Jun 23, 2009

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目都**請依題號順序依序寫在答案卷上，而是非與塡充題必須寫在第一頁**。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷上)

1. If $\sum_{n=1}^{\infty} a_n$ does not converge, then $\sum_{n=1}^{\infty} |a_n|$ does not converge.

2. Let $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla(xyz)$, then $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds = 0$ for any simple smooth closed-loop C in the space.

3. $\lim_{n\to\infty} \sqrt[n]{n} = \infty$.

4. The area of the fan-shaped region between $\theta=\alpha$, $\theta=\beta$ and the curve $r=f(\theta)$, $\alpha\le\theta\le\beta$, is $\int_\alpha^\beta \int_0^{f(\theta)} r\,dr\,d\theta$.

5. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent series, then $\sum_{n=1}^{\infty} (a_n+b_n)$ is also divergent.

6. $(\mathbf{i}-\mathbf{k})\times(\mathbf{j}+\mathbf{k}) = \mathbf{i}-\mathbf{j}+\mathbf{k}$.

7. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

8. The series $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n}}$ converges conditionally.

9. $y\,dx + x\,dy + 4\,dz$ is exact.

10. Let C be a counterclockwise simple closed curve in the xy-plane, and R be the region bounded by C. Then $\oint_C -y\,dx$ is equal to the area of R.

(下頁還有試題)

填充題 (*60* points)，每題 6 分。(請依題號順序依序寫在答案卷上)

1. Evaluate the integral $\int_C xy\,dx + (x+y)\,dy$ along the curve $y = x^2$ from $(-1,1)$ to $(2,4)$. Answer：__________.

2. Find the limit, $\lim_{(x,y)\to(0,0)} \dfrac{e^{2y}\sin x}{\sin 2x}$. Answer：__________.

3. Find the line integral of $f(x,y,z) = x+y+z$ over the straight-line segment from $(2,1,3)$ to $(0,1,-1)$. Answer：__________.

4. Evaluate $\int_{(0,0,0)}^{(3,3,1)} 2x\,dx - y^2\,dy - \dfrac{4}{1+z^2}\,dz$. Answer：__________.

5. Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and suppose that the region D is enclosed by the closed oriented surface S. Find the relation between $\iiint_D dV$ (the volume of D) and $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma$ (the flux of $\mathbf{F}$ outward through S) . Answer：__________.

6. Let $a_n = \dfrac{1}{n}\int_5^{2n} \dfrac{1}{x}\,dx$. Find the limit of the sequence $\{a_n\}$. Answer：______.

7. Let $f(x,y)$ be a continuous function. Write $\int_0^8 \int_{x^2}^{32\sqrt[3]{x}} f(x,y)\,dy\,dx$ as an iterated integral in the order $dx\,dy$. Answer：__________.

8. Find the curl of $\mathbf{F} = (x^2-y)\mathbf{i} + 4z\mathbf{j} + x^2\mathbf{k}$. Answer：__________.

9. Let $\begin{cases} u = x - 2y \\ v = x + y. \end{cases}$
Find the value of Jacobian $\partial(x,y)/\partial(u,v)$. Answer：__________.

10. Find the surface area of the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$. Answer：______.

計算問答證明題(*40* points)，每題 10 分。請依題號順序依序寫在答案卷上，可以用中文或英文作答。**請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points)

(a) Test the convergence or divergence for the series $\sum_{n=1}^{\infty} (-1)^n \dfrac{\ln n}{n - \ln n}$.

(b) Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \left(\dfrac{1+\sin(a_n)}{2}\right)^n$ converges.

(下頁還有試題)

2. (*10* points) Find the value of

$$\oint_C xy^2\,dx + (x^2y + 2x)\,dy,$$

where $C: \left\{(x, y) \in \mathbb{R}^2 : \dfrac{(x-10)^2}{4} + \dfrac{(y+21)^2}{9} = 1\right\}$.

3. (*10* points) Find the area of the surface $x^2 - 2\ln x + \sqrt{15}y - z = 0$ above the square R: $1 \le x \le 2$, $0 \le y \le 1$, in the xy-plane.

4. (*10* points) Evaluate

$$\iint_S \nabla \times (y\mathbf{i}) \cdot \mathbf{n}\,d\sigma,$$

where S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$.