微積分聯合教學　　先修班 段考二　　**Dec 2, 2008**

考試時間 120 分鐘，試題共十二題，滿分 120 分。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

1. (*10* points) Find the area of the surface generated by revolving the curve $r^2 = \cos 2\theta$ about the x-axis.

2. (*10* points) Which of the following are *not* always true? For each case, give a counter-example.

a. $|\mathbf{u}| = \sqrt{\mathbf{u}\cdot\mathbf{u}}$ **b.** $\mathbf{u}\cdot\mathbf{u} = |\mathbf{u}|$ **c.** $\mathbf{u}\times\mathbf{0} = \mathbf{0}$ **d.** $\mathbf{u}\times(-\mathbf{u}) = \mathbf{0}$

e. $\mathbf{u}\times\mathbf{v} = \mathbf{v}\times\mathbf{u}$ **f.** $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \mathbf{u}\times\mathbf{v} + \mathbf{u}\times\mathbf{w}$

g. $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v} = \mathbf{0}$ **h.** $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$

3. (*10* points) Find the curvature κ of the helix $\mathbf{r} = a\cos t\mathbf{i} + a\sin t\mathbf{j} + bt\mathbf{k}$ where $a, b > 0$. What is the largest value κ can have for a given value of b?

4. (*10* points) An object of mass m travels along the parabola $y = x^2$ with a constant speed of 10 units/sec. What is the force on the object due to its acceleration at $(0, 0)$ and at $(\sqrt{2}, 2)$? Write your answers in terms of $\mathbf{i}$ and $\mathbf{j}$.

5. (*10* points) Find the value of $\partial z/\partial x$ at the point $(1, 1, 1)$ if the equation

$$xy + z^3x - 2yz = 0$$

defines z as a function of the two independent variables x and y. Then find $\partial x/\partial z$ at the same point if the same equation defines x as a function of the two independent variables y and z.

6. (*10* points) In the following problems, find the derivative of the function at P in the direction of $\mathbf{w}$:

a. $f(x, y) = 2xy - 3y^2$, $P(5, 5)$, $\mathbf{w} = 4\mathbf{i} + 3\mathbf{j}$.

b. $g(x, y, z) = \cos xy + e^{yz} + \ln xz$, $P(1, 0, \frac{1}{2})$, $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

7. (*10* points) Find the linearization $L(x, y)$ of the function

$$f(x, y) = x^2 - 3xy + 5$$

at $P(2,1)$. Then find an upper bound for the magnitude $|E(x,y)|$ of the error in the approximation $f(x,y) \approx L(x,y)$ over the rectangle

$$R: \quad |x-2| \le 0.1, \quad |y-1| \le 0.1.$$

The error satisfies

$$|E(x,y)| \le \frac{1}{2}M(|x-x_0|+|y-y_0|)^2$$

if M is any upper bound for the values of $|f_{xx}|$, $|f_{xy}|$ and $f_{yy}|$ on R.

8. (*10* points) Let $T = g(x,y)$ be the temperature at the point (x,y) on the ellipse

$$x = 2\sqrt{2}\cos t, \quad y = \sqrt{2}\sin t, \quad 0 \le t \le 2\pi$$

and suppose that

$$\frac{\partial T}{\partial x} = y \quad \text{and} \quad \frac{\partial T}{\partial y} = x.$$

a. Locate the maximum temperatures on the ellipse by examining dT/dt and d^2T/dt^2.

b. Suppose that $T = xy - 2$, find the maximum values of T on the ellipse.

9. (*10* points) Find the critical point of

$$f(x,y) = xy + 2x - \ln x^2 y$$

in the open first quadrant ($x > 0$, $y > 0$) and show that f takes on a minimum there. Remember that the Hessian of f is

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

10. (*10* points) Find the largest product xyz the positive numbers x, y and z can have if they satisfy the equation $x + y + z^2 = 16$.

11. (*10* points) 寫出以下觀念的定義：

a. $f(x,y)$ is *continuous* at the point (x_0, y_0). **b.** $\left.\dfrac{\partial f}{\partial x}\right|_{(x_0,y_0)}$

c. The point (x_0, y_0) is a *critical point* of the function $f(x,y)$ in a region R.

12. (*10* points) In the following graphs, **0**–**9** show level curves for the functions graphed in **a**–**j**. Match each set of curves with the appropriate function.