微積分聯合教學　　先修班 期末考　　Dec 2, 2008

考試時間 120 分鐘，試題共十一題，有三頁，滿分 120 分。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

1. (*10* points) Write down the Taylor series for $\cos x$ at $x = 0$, and estimate the definite integral

$$\int_0^1 \cos x^2\, dx$$

with an error of magnitude no greater than 0.001.

2. (*10* points) Find the critical point of

$$f(x, y) = xy + 2x - \ln x^2 y$$

in the open first quadrant ($x > 0$, $y > 0$) and show that f takes on a minimum there. Remember that the Hessian of f is

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

3. (*10* points) Use Taylor's formula for

$$f(x, y) = \frac{1}{1 - x - y}$$

at the origin to find the quadratic approximation of f near the origin.

4. (*10* points) Find the average distance from a point $P(x, y)$ in the disk $x^2 + y^2 \leq a^2$ to the origin ($a > 0$).

5. (*10* points) Solve the system

$$u = 3x + 2y, \quad v = x + 4y$$

for x and y in terms of u and v. Then find the value of the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$. Then, use the above transformation to evaluate the double integral

$$\iint_R 3x^2 + 14xy + 8y^2\, dA$$

for the region R bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$, $y = -(1/4)x$, and $y = -(1/4)x + 1$.

6. (*10* points) Write down the definition for a vector field $\mathbf{F}$ being *conservative.* Let

$$\mathbf{F} = 3x^2\,\mathbf{i} + \frac{z^2}{y}\,\mathbf{j} + 2z\ln y\,\mathbf{k},$$

is $\mathbf{F}$ conservative? (State your reasons and show your work.) Evaluate the line integral

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds$$

along the line segment from $(1,1,1)$ to $(1,2,3)$.

7. (*10* points) Is the differential form $(6y + x)\,dx + (y + 2x)\,dy$ exact? (State your reasons and show your work.) Evaluate the integral

$$\oint_C (6y + x)\,dx + (y + 2x)\,dy$$

along the circle C: $(x-2)^2 + (y-3)^2 = 4$.

8. (*20* points) Let surface S be the cone

$$z = \sqrt{x^2 + y^2}, \quad 0 \le z \le 1.$$

(a) Find a parametrization of S.

(b) Find the surface area of S by the parametrization you found in part (a).

(c) Let C be the boundary of S: $x^2 + y^2 = 1$, $z = 1$. Let C be oriented counterclockwise as viewed from above. With this orientation, find the unit normal vector $\mathbf{n}$ for S.

(d) Find the circulation of the field $\mathbf{F} = (x^2 - y)\,\mathbf{i} + 4z\,\mathbf{j} + x^2\,\mathbf{k}$ around the curve C as described in part (c).

[請幫忙：這一題是 95 學年第二學期的期末考題，題目公布在微積分聯合教學網上。其實參考答案也找得到。如果同學曾經上網看過這一題，並且因此而做過練習，請在考卷上寫『有做過』；否則，請寫『沒做過』。在其他情況下準備過這一題的，也算是『沒做過』。這是為了要做教學成效研究而做的對照調查，請同學據實回答，我們絕不會因此扣分 (也不會加分)。謝謝。]

9. (*10* points) Let S be the portion of the cylinder $y = e^x$ in the first octant that projects parallel to the x-axis onto the rectangle R: $1 \le y \le 2$, $0 \le z \le 1$ in the yz-plane. Let $\mathbf{n}$ be the unit vector normal to S that points away from the yz-plane. Find the flux of the field $\mathbf{F}(x, y, z) = -2\mathbf{i} + 2y\,\mathbf{j} + z\,\mathbf{k}$ across S in the direction of $\mathbf{n}$.

10. (*10* points) Let C be a simple closed smooth curve in the plane $2x + 2y + z = 2$, oriented as shown on the right. Show that

$$\oint_C 2y\,dx + 3z\,dy - x\,dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C.

11. (*10* points) Among all rectangular solids defined by the inequalities $0 \le x \le a$, $0 \le y \le b$, and $0 \le z \le 1$, find the one for which the total flux of $\mathbf{F} = (-x^2 - 4xy)\,\mathbf{i} - 6yz\,\mathbf{j} + 12z\,\mathbf{k}$ outward through the six sides is greatest. What *is* the greatest flux? (Needless to say that a and b are positive numbers.)

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