

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷第一頁上)

1. If an odd function $g(x)$ has a local minimum value at $x = c$, then g assumes a local maximum at the point $x = -c$.
2. The graph of every polynomial of even degree has at least one horizontal tangent.
3. If $f(x)$ is differentiable and $f'(x) = 0$ has no solutions, then $f(x)$ has no local minima or maxima.
4. If $f(x)$ is continuous but has no critical points in $[0, 1]$, then $f(x)$ has no absolute min or absolute max on $[0, 1]$.
5. Suppose that $F'(x) = f(x)$ and $G'(x) = g(x)$. If f and g differ by a constant, then $F = G$.
6. If y is a function of x , then $\frac{d}{dx} \sin(y^2) = 2y \cos y^2$.
7. There is an antiderivative for every continuous function.
8. If $f''(c) = 0$, then c is an inflection point of $f(x)$.
9. Let $f(x)$ be a continuous function on $[a, b]$, and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$. Then $F(x)$ is differentiable on (a, b) .
10. Suppose $f(x)$ is differentiable and c is a critical point of $f(x)$. If $f''(c) = 0$, then by the Second derivative test, $f(c)$ is not a local extremum.

(下頁還有試題)

填充題 (40 points), 每題 5 分。(請依題號順序依序寫在答案卷第一頁上)

1. Evaluate the definite integral $\int_0^{\pi/4} \tan^2 x \sec^2 x \, dx$. Answer : _____.
2. Find $\frac{dy}{dt}$, where $\sin(ty) = \frac{t^2}{y}$. Answer : _____.
3. Find the values $a \leq b$ such that $\int_a^b (x^2 - 9) \, dx$ has minimal value.
Answer : _____.
4. Find the derivative of $y = \sqrt{1 + \sqrt{1 + \sqrt{2x}}}$. Answer : _____.
5. Evaluate $\int \frac{\cos 2x}{(1 + \sin 2x)^2} \, dx$. Answer : _____.
6. $f(x) = \begin{cases} \frac{1}{2}x^4, & \text{if } x \text{ is an integer,} \\ \sin x \cos x, & \text{if } x \text{ is not an integer.} \end{cases}$ Find $\int_0^{10} f(x) \, dx$.
Answer : _____.
7. Find the linearization of $g(x) = 5 + \int_1^{x^2} \sec(t-1) \, dt$ at $x = -1$.
Answer : _____.
8. Find the maximum value of the function $f(x) = \frac{3x}{x^2 + 1}$ on the interval $[0, 4]$.
Answer : _____.

(下頁還有試題)

計算問答證明題(60 points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

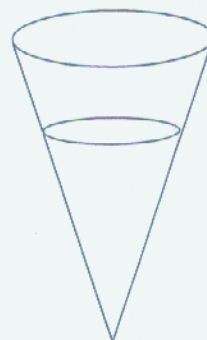
1. (10 points) Express the limit as an integral and evaluate.

$$\lim_{N \rightarrow \infty} \frac{\pi}{N} \sum_{j=0}^{N-1} \sin\left(\frac{\pi}{2} + \frac{\pi j}{N}\right).$$

2. (10 points) Show that $f(x) = x + \frac{x}{x^2 + 1}$ has **precisely one** real root.

3. (10 points) Find the area of the region under the graph of $f(x) = \frac{x^2 + 2x}{(x^3 + 3x^2 + 1)^2}$ over $[1, 2]$.

4. (10 points) Water pours into a conical tank (see the figure below) of height $8ft$ and radius $4ft$ at a rate of $6ft^3/min$. How fast is the water level rising when it is $3ft$ high? ($V = \frac{1}{3}\pi hr^2$)



5. (10 points) Design a cylindrical can of volume $20ft^3$ so that it uses the least amount of metal. In other words, minimize the surface area of the can (including its top and bottom). Please justify your answer.
6. (10 points) Consider the function $f(x) = \frac{2}{x^2 - 1}$.
- Determine the intervals on which $f(x)$ is increasing or decreasing.
 - Determine the intervals on which $f(x)$ is concave up or down.
 - Sketch the graph of the function. Indicate the asymptotes, local extrema, and points of inflection.

(試題結束)