1. If you are given both the doubling time and the growth constant of a quantity that increases exponentially, then you can determine the initial amount.

2. If \( k > 0 \), then all solutions of \( y' = -k(y-b) \) approach the same limit as \( t \to \infty \).

3. If \( f(x) = 2^x \), then \( f'(x) = x2^{x-1} \).

4. We may use L'Hopital’s Rule to get \( \lim_{x \to 1} \frac{x^2 + 1}{2x + 1} = \lim_{x \to 1} \frac{2x}{2} = 1 \).

5. Let \( P(x)/Q(x) \) be a proper rational function, where \( Q(x) \) factors as a product of distinct linear factors \( (x - a_i) \). Then \( \int \frac{P(x)}{Q(x)} \, dx \) is a sum of logarithmic terms \( A_i \ln |(x - a_i)| \) for some constants \( A_i \).

6. The Present Value of \( N \) dollars received at time \( T \) is the amount you would have to invest today in order to receive \( N \) dollars at time \( T \).

7. If \( f(x) = x^2 \) then the average value of \( f \) on \([0,2]\) is \( 4/3 \).

8. \( f(x) = (\ln x)^2 \) grows faster than \( g(x) = \sqrt{x} \) as \( x \to \infty \).

9. The integral \( \pi \int_a^b [f(x) - g(x)]^2 \, dx \) expresses the volume of the solid obtained by rotating the area between \( y = f(x) \) and \( y = g(x) \) over \([a,b]\) around the \( x \)-axis. (Assume \( f(x) \geq g(x) \geq 0 \))

10. If \( f(x) \) is strictly increasing, then \( f^{-1}(x) \) is strictly increasing.

(下頁還有試題)
1. Evaluate \( \int \frac{\cos 2x}{(1 + \sin 2x)^2} \, dx \). Answer : 

2. Find \( \lim_{x \to 0} \left( \cos \frac{1}{x} \right) \left( \sin x \right) \). Answer : 

3. Find the area between the graph of \( y = \sin x \) and \( y = 1 - \cos x \) over the interval \( -\frac{\pi}{2} \leq x \leq 0 \). Answer : 

4. Let \( g(x) \) be the inverse of \( f(x) = 2x^3 + 3x + 3 \). Find \( g'(8) \).

Answer :

5. Evaluate \( \lim_{x \to \infty} \left( \frac{x}{x + 1} \right)^x \). Answer :

6. Find \( \lim_{x \to 0} \frac{1}{3x^2} \int_{x^2}^{0} \sin \left( t + \frac{\pi}{2} \right) \, dt \). Answer :

7. Use Integration by Parts to evaluate \( \int e^x \sin x \, dx \). Answer :

8. Evaluate \( \int_0^1 e^{\sqrt{x}} \, dx \). Answer :

(下頁還有試題)
1. (10 points) Evaluate the limit.
   a. \( \lim_{x \to 0} \left( \cot x - \frac{1}{x} \right) \).
   b. \( \lim_{x \to 0} x \sin x. \)

2. (10 points) Find the volume of the solid obtained by rotating region enclosed by
   \( x = 0, y = 4 \) and \( y = x^2 + 1 \) about the \( x \)-axis and the \( y \)-axis.

3. (10 points) Evaluate the integral.
   a. \( \int \frac{1}{(x^2 + 2)^2} \, dx. \)
   b. \( \int \frac{4 - x}{x(x^2 + 2)^2} \, dx. \)

4. (10 points) Find the tangent line to the curve
   \[ x^2 \cos^2 y = \sin y \]
   at the point \((0, \pi)\).

5. (10 points) Let \( f(x) = x^{1/x} \) in the domain \( \{ x : x > 0 \} \).
   a. Calculate \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to \infty} f(x). \)
   b. Find the minimum value of \( f(x) \) on \((0, \infty)\).

6. (10 points) Find the following integrals respectively.
   a. \( \int \cos^3(\pi \theta) \sin^4(\pi \theta) \, d\theta. \)
   b. \( \int \frac{1}{\sqrt{x^2 - 4x + 8}} \, dx. \)