微積分聯合教學　段考三　**Jan 12, 2010**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(請依題號順序依序寫在答案卷第一頁上)

1. If you are given both the doubling time and the growth constant of a quantity that increases exponentially, then you can determine the initial amount.

2. If $k > 0$, then all solutions of $y' = -k(y-b)$ approach the same limit as $t \to \infty$.

3. If $f(x) = 2^x$, then $f'(x) = x2^{x-1}$.

4. We may use L'Hopital's Rule to get $\lim_{x\to 1} \dfrac{x^2+1}{2x+1} = \lim_{x\to 1} \dfrac{2x}{2} = 1$.

5. Let $P(x)/Q(x)$ be a proper rational function, where $Q(x)$ factors as a product of distinct linear factors $(x - a_i)$. Then $\displaystyle\int \frac{P(x)}{Q(x)}\, dx$ is a sum of logarithmic terms $A_i \ln|(x - a_i)|$ for some constants A_i.

6. The Present Value of N dollars received at time T is the amount you would have to invest today in order to receive N dollars at time T.

7. If $f(x) = x^2$ then the average value of f on $[0, 2]$ is $4/3$.

8. $f(x) = (\ln x)^2$ grows faster than $g(x) = \sqrt{x}$ as $x \to \infty$.

9. The integral $\pi \displaystyle\int_a^b [f(x) - g(x)]^2\, dx$ expresses the volume of the solid obtained by rotating the area between $y = f(x)$ and $y = g(x)$ over $[a, b]$ around the $x-$axis. (Assume $f(x) \geq g(x) \geq 0$)

10. If $f(x)$ is strictly increasing, then $f^{-1}(x)$ is strictly increasing.

(下頁還有試題)

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷第一頁上)

1. Evaluate $\int \dfrac{\cos 2x}{(1+\sin 2x)^2}\, dx$. Answer ：_____.

2. Find $\lim\limits_{x\to 0}\left[\left(\cos\dfrac{1}{x}\right)(\sin x)\right]$. Answer ：_____.

3. Find the area between the graph of $y=\sin x$ and $y=1-\cos x$ over the interval $-\dfrac{\pi}{2}\le x\le 0$. Answer ：_____.

4. Let $g(x)$ be the inverse of $f(x)=2x^3+3x+3$. Find $g'(8)$.

Answer ：_____.

5. Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{x}{x+1}\right)^x$. Answer ：_____.

6. Find $\lim\limits_{x\to 0}\dfrac{1}{3x^2}\displaystyle\int_{x^2}^{0}\sin\left(t+\frac{\pi}{2}\right)dt$. Answer ：_____.

7. Use Integratioin by Parts to evaluate $\int e^x\sin x\, dx$. Answer ：_____.

8. Evaluate $\displaystyle\int_0^1 e^{\sqrt{x}}\, dx$. Answer ：_____.

(下頁還有試題)

計算問答證明題(*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Evaluate the limit.

a. $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$.

b. $\lim_{x\to 0} x^{\sin x}$.

2. (*10* points) Find the volume of the solid obtained by rotating region enclosed by $x = 0$, $y = 4$ and $y = x^2 + 1$ about the $x-$axis and the $y-$axis.

3. (*10* points) Evaluate the integral.

a. $\int \frac{1}{(x^2+2)^2}\, dx$.

b. $\int \frac{4-x}{x(x^2+2)^2}\, dx$.

4. (*10* points) Find the tangent line to the curve

$$x^2\cos^2 y = \sin y$$

at the point $(0, \pi)$.

5. (*10* points) Let $f(x) = x^{1/x}$ in the domain $\{x : x > 0\}$.

a. Calculate $\lim_{x\to 0+} f(x)$ and $\lim_{x\to\infty} f(x)$.

b. Find the minimum value of $f(x)$ on $(0, \infty)$.

6. (*10* points) Find the following integrals respectively.

a. $\int \cos^3(\pi\theta)\sin^4(\pi\theta)\, d\theta$.

b. $\int \frac{1}{\sqrt{x^2-4x+8}}\, dx$.

(試題結束)