微積分聯合教學　　九八先修班段考一　　Oct 20, 2009

考試時間 120 分鐘，試題共十二題，滿分 120 分。試場依聯合教學之會考規則管理。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

1. (*10* points) (a) Calculate the fluid force on one side of the plate (an isosceles triangle) shown in the following figure.

(b) Calculate the length of the astroid $x^{2/3} + y^{2/3} = 1$.

2. (*10* points) (a) Show that the *harmonic series* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(b) Determine the convergence or divergence of

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3 - n^2}}.$$

3. (*10* points) (a) Expand $f(x) = \dfrac{1}{1+3x^7}$ in a power series with center $c = 0$ and determine the set of x for which the expansion is valid.

(b) Use the Maclaurin expansion for e^{-t^2} to express

$$\int_0^x e^{-t^2}\, dt$$

as an alternating power series in t. How many terms of the infinite series are needed to approximate the integral for $x = 1$ to within an error of at most 0.001?

4. (*10* points) The Leibniz Test cannot be applied to

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots.$$

Why not? Show that it converges by another method.

5. (*10* points) Let $\{a_n\}$ be the sequence

$$\sqrt{2}, \quad \sqrt{2\sqrt{2}}, \quad \sqrt{2\sqrt{2\sqrt{2}}}, \quad \ldots.$$

Show that $\{a_n\}$ is increasing with an upper bound 2. Then find the limit of $\{a_n\}$.

6. (*10* points) Find the centroid of the top half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

for arbitrary a, $b > 0$.

7. (*10* points) Find the surface area of the ellipsoid obtained by rotating the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about the x-axis, where a, b are arbitrary positive constants.

8. (*10* points) (a) Find the points on the curve $c(t) = (t^2 - 9, t^2 - 8t)$ where the tangent has slope $1/2$.

(b) Find the minimum speed of a particle with trajectory $c(t) = (t^3 - 4t, t^2 + 1)$ for $t \geq 0$. [Hint: It is easier to find the minimum of the square of the speed.]

9. (*10* points) Find the values of x for which the following power series converge.

$$\text{(a)} \sum_{n=2}^{\infty} \frac{x^n}{\ln n}, \quad \text{(b)} \sum_{n=1}^{\infty} \frac{(x-4)^n}{n^4}.$$

10. (*10* points) Find the first five terms of the Maclaurin series for

$$f(x) = \frac{\sin x}{1 - x}.$$

11. (*10* points) Find the Maclaurin polynomial of degree n for $f(x) = \sin^{-1} x$ for an odd integer n.

12. (*10* points) Why is it *impossible* to expand $f(x) = |x|$ as a power series that converges in an interval around $x = 0$?