1. (10 points) (a) Calculate the fluid force on one side of the plate (an isosceles triangle) shown in the following figure.
   (b) Calculate the length of the astroid \( x^{2/3} + y^{2/3} = 1 \).

2. (10 points) (a) Show that the harmonic series \( \sum_{n=1}^{\infty} \frac{1}{n} \) is divergent.
   (b) Determine the convergence or divergence of
   \[
   \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3 - n^2}}.
   \]

3. (10 points) (a) Expand \( f(x) = \frac{1}{1 + 3x^7} \) in a power series with center \( c = 0 \) and determine the set of \( x \) for which the expansion is valid.
   (b) Use the Maclaurin expansion for \( e^{-t^2} \) to express
   \[
   \int_{0}^{a} e^{-t^4} \, dt
   \]
   as an alternating power series in \( t \). How many terms of the infinite series are needed to approximate the integral for \( x = 1 \) to within an error of at most 0.001?

4. (10 points) The Leibniz Test cannot be applied to
   \[
   \frac{1}{2} - \frac{1}{3} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{2^3} - \frac{1}{3^3} + \cdots.
   \]
   Why not? Show that it converges by another method.
5. (10 points) Let \( \{a_n\} \) be the sequence

\[
\sqrt{2}, \quad \sqrt{2\sqrt{2}}, \quad \sqrt{2\sqrt{2\sqrt{2}}}, \ldots.
\]

Show that \( \{a_n\} \) is increasing with an upper bound 2. Then find the limit of \( \{a_n\} \).

6. (10 points) Find the centroid of the top half of the ellipse

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

for arbitrary \( a, b > 0 \).

7. (10 points) Find the surface area of the ellipsoid obtained by rotating the ellipse

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

about the \( x \)-axis, where \( a, b \) are arbitrary positive constants.

8. (10 points) (a) Find the points on the curve \( c(t) = (t^2 - 9, t^2 - 8t) \) where the tangent has slope 1/2.

(b) Find the minimum speed of a particle with trajectory \( c(t) = (t^3 - 4t, t^2 + 1) \) for \( t \geq 0 \). [Hint: It is easier to find the minimum of the square of the speed.]

9. (10 points) Find the values of \( x \) for which the following power series converge.

(a) \( \sum_{n=2}^{\infty} \frac{x^n}{\ln n} \), \quad (b) \( \sum_{n=1}^{\infty} \frac{(x - 4)^n}{n^4} \).

10. (10 points) Find the first five terms of the Maclaurin series for

\[
f(x) = \frac{\sin x}{1 - x}.
\]

11. (10 points) Find the Maclaurin polynomial of degree \( n \) for \( f(x) = \sin^{-1} x \) for an odd integer \( n \).

12. (10 points) Why is it impossible to expand \( f(x) = |x| \) as a power series that converges in an interval around \( x = 0 \)?