微積分聯合教學先修班　段考二　**Dec 01, 2009**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*20* points)，請答 **T** (True) 或 **F** (False)。每題 4 分。(請依題號順序依序寫在答案卷第一頁上)

1. If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then it is impossible that $\|\mathbf{u}+\mathbf{v}\| = \|\mathbf{u}\| = \|\mathbf{v}\|$.

2. $\mathbf{r}_1(t) = \left\langle t, -t^3, 3t^2-2\right\rangle$ collides with $\mathbf{r}_2(t) = \left\langle 4t+6, 2t^2, 8-t\right\rangle$.

3. The arc length parametrization of $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ is $\mathbf{r}_1(s) =< 3\cos\frac{s}{3}$, $3\sin\frac{s}{3}, \frac{s}{3} >$.

4. The level curve $f(x,y) = c$ is the projection of the horizontal trace at height c to the yz-plane.

5. For any polynomial function f of x and y, $f_{xy} = f_{yx}$.

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷第一頁上)

1. Find the unit vector $\mathbf{X}$ such that $\langle 1,1,1\rangle \times \mathbf{X} = \langle 1,-1,0\rangle$.
Answer ：___________.

2. Find the equation of the plane contains the lines $\mathbf{r}_1(t) = \langle 3t, 2t, t\rangle$ and $\mathbf{r}_2(t) = \langle t, t, t\rangle$. Answer ：___________.

(下頁還有試題)

3. Let L be the intersection of the plane $x + y - z = 3$ and $2x + 3y - z = 2$. Find the parametric equations for the line. Answer ： ___________.

4. Let $\mathbf{r}(t) = \left\langle 1 + 2t, t^2, \frac{1}{3}t^3 \right\rangle$. Find $N(1)$. Answer ： ___________.

5. Find an arc length parametrization of the circle in the plane $z = 9$ with radius 4 and center $(1, 4, 9)$. Answer ： ___________.

6. Find the point on the plane $x + 2y + 3z = 4$ which is closest to the origin. Answer ： ___________.

7. Find an equation of the tangent plane of $G(u, w) = \sin uw$ at the point $\left(\frac{\pi}{6}, 1\right)$. Answer ： ___________.

8. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$. Answer ： ___________.

計算問答證明題(*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) If $r = \cos\theta + \sin\theta$, $x = r\cos\theta$ and $y = r\sin\theta$. Find $\frac{dy}{dx}|_{\theta=\frac{\pi}{4}}$.

2. (*10* points) Let $r = 2\cos\theta - \sqrt{3}$.

a. Plot the curve.

b. Find the area of the region between the inner and outer loop.

3. (*10* points) At a certain moment, a moving particle has velocity $\mathbf{v} = \langle 2, -1, 2 \rangle$ and $\mathbf{a} = \langle 0, 3, 4 \rangle$. Find $\mathbf{T}$, $\mathbf{N}$, and the decomposition of $\mathbf{a}$ into tangential and normal components.

(下頁還有試題)

4. (*10* points) Find $\mathbf{T}(t)$, $\mathbf{N}(t)$ and $\mathbf{K}(t)$ for

$$\mathbf{r}(t) = (\cos t + t \sin t)\,\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j}, \quad t > 0.$$

5. (*10* points) Find a parametrization of the osculating circle of $y = x^2$ at $x = 1$.

6. (*10* points) Suppose $\mathbf{r} = \mathbf{r}(t)$ lies on a sphere of radius R for all t, $\mathbf{r}$, $\mathbf{r}'$ and $\mathbf{r} \times \mathbf{r}'$ forms a right-handed system. Show that

a. $\mathbf{r} \perp \mathbf{r}'$.

b. $\mathbf{r}' = \dfrac{(\mathbf{r} \times \mathbf{r}') \times \mathbf{r}}{R^2}$.

(試題結束)