微積分聯合教學 九八暑期 週考二  July 21, 2009

考試時間 50 分鐘，試題共五題，滿分 50 分。請在考試卷上以中文或英文
盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必複題。
考試卷務必寫學號、姓名，試題不必繳回。

1. (10 points) (a) Draw the graph of \( f(x) = x - [x] \), where \( [x] \) is the greatest integer
   function.
   (b) What is the domain of \( f(x) \)?
   (c) What is the range of \( f(x) \)?
   (d) At which points is \( f \) discontinuous?

2. (10 points) Evaluate the following limits.
   (a) \( \lim_{h \to 0} \frac{1}{(h+2)^2} - \frac{1}{4} \)
   (b) \( \lim_{\theta \to 0} \frac{\cos \theta - 1}{\sin \theta} \)
   (c) \( \lim_{t \to 0} \frac{\sin^2 t}{t} \)
   (d) \( \lim_{x \to 0} \frac{\tan 4x}{\tan 9x} \)

3. (10 points) (a) Use the Squeeze Theorem to prove that
   \[ \lim_{x \to c} |f(x)| = 0 \implies \lim_{x \to c} f(x) = 0 \]
   (b) Evaluate the limit
   \[ \lim_{x \to 0} \cos \left( \frac{1}{x} \right) \sin x \]

4. (10 points) Let \( f(x) \) be the function whose graph is shown in the following figure.
   (a) Determine \( f'(a) \) for \( a = 1, 2, 4, 7 \).
   (b) Which is larger: \( f'(5.5) \) or \( f'(6.5) \)?
   (c) Show that \( f'(3) \) does not exist.

(背面還有)
5. (10 points) The definition of \[
\lim_{x \to c} f(x) = L
\]
is that \(\forall \epsilon > 0, \exists \delta > 0\) such that
\[
0 < |x - c| < \delta \implies |f(x) - L| < \epsilon
\]
(a) Write down the definition for
\[
\lim_{x \to c} f(x) \neq L
\]
(b) Prove rigorously that
\[
\lim_{x \to 0} \frac{|x|}{x} \neq 0
\]