微積分聯合教學　　九八暑期 週考二　　July 21, 2009

考試時間 50 分鐘，試題共五題，滿分 50 分。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

1. (*10* points) (a) Draw the graph of $f(x) = x - [x]$, where $[x]$ is the *greatest integer function.*

(b) What is the domain of $f(x)$?

(c) What is the range of $f(x)$?

(d) At which points is f discontinuous?

2. (*10* points) Evaluate the following limits.

(a) $\lim_{h\to 0} \dfrac{\dfrac{1}{(h+2)^2} - \dfrac{1}{4}}{h}$

(b) $\lim_{\theta\to 0} \dfrac{\cos\theta - 1}{\sin\theta}$

(c) $\lim_{t\to 0} \dfrac{\sin^2 t}{t}$

(d) $\lim_{x\to 0} \dfrac{\tan 4x}{\tan 9x}$

3. (*10* points) (a) Use the Squeeze Theorem to prove that

$$\lim_{x\to c} |f(x)| = 0 \quad \Longrightarrow \quad \lim_{x\to c} f(x) = 0$$

(b) Evaluate the limit

$$\lim_{x\to 0} \cos(\frac{1}{x}) \sin x$$

4. (*10* points) Let $f(x)$ be the function whose graph is shown in the following figure.

(a) Determine $f'(a)$ for $a = 1, 2, 4, 7$.

(b) Which is larger: $f'(5.5)$ or $f'(6.5)$?

(c) Show that $f'(3)$ does not exist.

(背面還有)

5. (*10* points) The *definition* of

$$\lim_{x \to c} f(x) = L$$

is that $\forall \epsilon > 0$, $\exists \delta > 0$ such that

$$0 < |x - c| < \delta \quad \Longrightarrow \quad |f(x) - L| < \epsilon$$

(a) Write down the definition for

$$\lim_{x \to c} f(x) \neq L$$

(b) Prove rigorously that

$$\lim_{x \to 0} \frac{|x|}{x} \neq 0$$