# 微積分聯合教學　　九八暑期 週考七　　Aug 25, 2009

考試時間 50 分鐘，滿分 50 分。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

**1.** (*20* points) Calculate the following definite or indefinite integrals.

(a) $\displaystyle\int_{2/\sqrt{3}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}}$　　(b) $\displaystyle\int_0^2 \frac{dx}{x^2+4}$

(c) $\displaystyle\int \frac{1+x}{\sqrt{1-x^2}}\,dx$　　(d) $\displaystyle\int \frac{\tan^{-1}x}{x^2+1}\,dx$

(e) $\displaystyle\int_0^3 \frac{x}{x^2+9}\,dx$

**2.** (*10* points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 4}\left[\frac{1}{\sqrt{x}-2}-\frac{4}{x-4}\right]$　　(b) $\displaystyle\lim_{x\to 1}\frac{e^x-e}{\ln x}$

(c) $\displaystyle\lim_{x\to 0^+}\sqrt{x}\ln x$　　(d) $\displaystyle\lim_{t\to\infty}\frac{\ln(t+2)}{\log_2 t}$

**3.** (*10* points) Let $f(x)=x^{1/x}$ be a function in the domain $x>0$. Find

$$\lim_{x\to 0^+} f(x) \quad \text{and} \quad \lim_{x\to\infty} f(x).$$

Find the critical points of $f(x)$ and determine the local extrema for $f(x)$. Suppose we know that

$$\lim_{x\to 0^+} f'(x)=0,$$

sketch the graph of $y=f(x)$ in its domain.

**4.** (*10* points) A painting of length $b$ is located at a height $h$ above eye level. Find the distance $x$ at which the viewing angle $\theta$ is maximized.