

考試時間 120 分鐘，試題共十二題，有兩頁，滿分 120 分。請在考試卷上以中文或英文盡量依序作答，請詳列計算過程，否則不予計分。需標明題號但不必抄題。考試卷務必寫學號、姓名，試題不必繳回。

1. (10 points) Calculate the following integrals.

$$(a) \int \frac{dx}{(x^2 + 9)^2} \qquad (b) \int \frac{x^2}{\sqrt{9 - x^2}} dx$$

2. (10 points) Calculate the following integrals.

$$(a) \int_0^{\infty} e^{-x} \cos x dx \qquad (b) \int_0^1 \frac{\ln x}{x^2} dx$$

3. (10 points) Find the derivatives of the following functions.

$$(a) y = \frac{\sin^{-1} x}{\ln x} \qquad (b) y = x^{\sqrt{x} + \ln x}$$

4. (10 points) Find the following limits.

$$(a) \lim_{t \rightarrow \infty} \frac{\ln(t+2)}{\log_2 t} \qquad (b) \lim_{x \rightarrow 0} \frac{\sinh x^2}{\cosh x - 1}$$

5. (10 points) Let S be the solid obtained by rotating the region below the graph of $y = 1/x$ about the x -axis for $1 \leq x < \infty$. Find the volume of S . The surface area of S is

$$A = 2\pi \int_1^{\infty} x^{-1} \sqrt{1 + x^{-4}} dx,$$

show that A is infinite.

6. (10 points) Recall the basic formulae:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \sin \alpha \sin \beta \mp \cos \alpha \cos \beta$$

Prove that, for any integers m, n with $m \neq \pm n$:

$$\int_0^{2\pi} \sin mx \cos nx dx = 0, \quad \int_0^{\pi} \sin mx \sin nx dx = 0, \quad \int_0^{\pi} \cos mx \cos nx dx = 0$$

7. (10 points) Find the solutions to $y' = 4(y - 12)$ satisfying $y(0) = 20$ and $y(0) = 0$, and sketch their graphs.

8. (10 points) Let $f(x) = xe^{-x}$, show that $f(x)$ has an inverse on $[1, \infty)$. Let $g(x)$ be this inverse. Find the domain and range of $g(x)$ and compute $g'(2e^{-2})$.
9. (10 points) Water is pumped into a spherical tank of radius 5 ft from a source located 2 ft below a hole at the bottom. Calculate the work $W(x)$ required to fill the tank to height x ft from the bottom of the sphere, $0 \leq x \leq 10$. The density of water is 64.2 lb/ft^3 .
10. (10 points) A particle starts at the origin at time $t = 0$ (in seconds) and moves with velocity $v(t)$ as shown in the following figure.
- How many times does the particle return to the origin in the first 12 seconds?
 - Where is the particle located at time $t = 12$?
 - At which time t does the particle's distance to the origin reach the maximum?
11. (10 points) For which values of α is $f(x) = |x|^\alpha$ differentiable at $x = 0$?
12. (10 points) Let $f(x) = x \lfloor 1/x \rfloor$ where $\lfloor x \rfloor$ is the greatest integer function.
- Sketch the graph of $f(x)$ on the interval $[1/4, 2]$.
 - Show that, for $x \neq 0$:

$$\frac{1}{x} - 1 < \left\lfloor \frac{1}{x} \right\rfloor \leq \frac{1}{x}$$

- Prove that

$$\lim_{x \rightarrow 0} f(x) = 1.$$

(短暫的) 暑假快樂；請注意交通安全，避免流感