

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

- T** 1. If both $f(x)$ and $g(x)$ are increasing functions and $(f \circ g)(x)$ is well defined, then $(f \circ g)(x)$ is also increasing.

$$\begin{aligned}x_1 < x_2 &\Rightarrow g(x_1) < g(x_2) && (g \text{ is increasing}) \\&\Rightarrow f(g(x_1)) < f(g(x_2)) && (f \text{ is increasing}) \\&\Rightarrow (f \circ g)(x_1) < (f \circ g)(x_2)\end{aligned}$$

- T** 2. There is an antiderivative for every continuous function.

This is true because of FCTI.

- F** 3. If c is a critical number of a function $f(x)$, then $f(x)$ has a local maximum or minimum at c .

*Consider $f(x) = x^3$, then $f'(x) = 3x^2$.
Note that $x=0$ is a critical number,
but $f(0) = 0$ is not a local extremum.*

4. $x^{\frac{1}{\ln x}} = 2$ has no solution.

T Take logarithm to both sides,

$$\frac{1}{\ln x} \cdot \ln x = \ln 2,$$

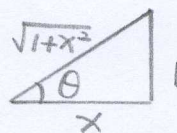
but $1 \neq \ln 2$

\therefore No solution.

5. $\cos(\tan^{-1}(\sin(\cot^{-1}x))) = \sqrt{\frac{x^2+1}{x^2+2}}$

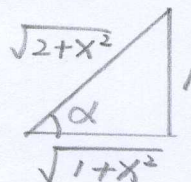
T Let $\theta = \cot^{-1}x$, $\cot \theta = x$

$$\sin \theta = \frac{1}{\sqrt{1+x^2}}$$



Let $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, $\tan \alpha = \frac{1}{\sqrt{1+x^2}}$

$$\cos \alpha = \frac{\sqrt{x^2+1}}{x^2+2}$$



6. Suppose f is one-to-one, twice differentiable function with inverse function. If f is decreasing and concave upward, then its inverse function is concave downward.

F

Let g be the inverse function of f

$$g'(x) = \frac{1}{f'(g(x))} = [f'(g(x))]^{-1}$$

$$g''(x) = -[f'(g(x))]^{-2} \cdot f''(g(x)) \cdot g'(x)$$

$$= -\frac{f''(g(x))}{[f'(g(x))]^3}$$

f is decreasing, $\therefore f' < 0$.

f is concave upward, $f'' > 0$.

$$\Rightarrow g''(x) > 0.$$

The inverse function is concave upward.

