先修班　段考一　　**Jul 15, 2010**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。(請依題號順序依序寫在答案卷第一頁上)

1. If f is a function, then $f(3x) = 3f(x)$.

2. If $x_1 < x_2$ and f is an increasing function, then $f(x_1) > f(x_2)$.

3. If f and g are functions, then $f \circ g = g \circ f$.

4. If $f(s) = f(t)$, then $s = t$.

5. $\lim_{x\to 4}\left(\frac{2x}{x-4} - \frac{8}{x-4}\right) = \lim_{x\to 4}\frac{2x}{x-4} - \lim_{x\to 4}\frac{8}{x-4}$

6. If $\lim_{x\to 6}[f(x)g(x)]$ exists, then the limit must be $f(6)g(6)$.

7. If p is a polynomial, then $\lim_{x\to b} p(x) = p(b)$.

8. If f a continuous at 5 and $f(5) = 2$ and $f(4) = 3$, then $\lim_{x\to 2} f(4x^2 - 11) = 2$.

9. Let f be a function such that $\lim_{x\to 0} f(x) = 6$. Then there exists a number δ such that if $0 < |x| < \delta$, then $|f(x) - 6| < 1$.

10. If $\lim_{x\to 0} f(x) = -\infty$ and $\lim_{x\to 0} g(x) = \infty$, then $\lim_{x\to 0}[f(x) + g(x)] = 0$

(下頁還有試題)

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷第一頁上)

1. The table shows the position of a cyclist.

Find the average velocity for the time period $[3, 5]$. Answer ：___________.

2. Let

$$f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0 \\ 3 - x \quad , & \text{if } 0 \le x < 3 \\ (x-3)^2 \quad , & \text{if } x > 3 \end{cases}$$

Evaluate $\lim_{x \to 0} f(x)$, if it exists.

Answer ：___________.

3. Evaluate $\lim_{x \to -1} \dfrac{x-2}{x^2+4x-3}$. Answer ：___________.

4. If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, find $\lim_{x \to a} [f(x)g(x)]$.

Answer ：___________.

5. Let $f(x) = 2x - 1$, $g(x) = x^2$, and $h(x) = 1 - x$. Find $f \circ g \circ h$.

Answer ：___________.

6. Determine the infinite limit, $\lim_{x \to 1^+} \dfrac{x+1}{x \sin \pi x}$. Answer ：___________.

7. Find the domain of $\dfrac{f}{g}$ if $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-1}$.

Answer ：___________.

8. Determine which equation to match the following graph.

a. $y = 3x$ **b.** $y = 3^x$ **c.** $y = x^3$ **d.** $y = \sqrt[3]{x}$

Answer ：___________.

(下頁還有試題)

計算問答證明題(*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Prove the statement using the ϵ, δ definition of limit.

$$\lim_{x \to 3} (x^2 + x - 4) = 8$$

2. (*10* points)

$$\sin x = 2 - x$$

a. Prove the function is continuous.

b. Prove that the equation has at least one real root.

3. (*10* points) Evaluate the limit, if it exists.

$$\lim_{x \to 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x}$$

(*Hint*: Use the squeeze theorem.)

4. (*10* points) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & \text{if } x < 2 \\ ax^2 - bx + 3 \quad , & \text{if } 2 \leq x < 3 \\ 2x - a + b \quad , & \text{if } x \geq 3 \end{cases}$$

5. (*10* points) Evaluate $\lim_{x \to 0} \dfrac{|2x - 1| - |2x + 1|}{x}$.

(試題結束)

99學年暑期微積分先修班第一次段考解答

1 是非題

1.F 2.F 3.F 4.F 5.F 6.F 7.T 8.T 9.T 10.F

2 填充題

1. 7.55 (m/s) 2. 極限不存在 3. $\frac{1}{2}$ 4. $\frac{3}{4}$ 5. $2x^2-4x+1$ 6. $-\infty$ 7. $(-\infty,-1)\cup(1,3]$ 8. d

3 計算證明題

1. Given $\epsilon>0$. If $|x-3|<\delta$ for some δ, $0<\delta<1$, then

$$|(x^2+x-4)-8|=|x^2+x-12|=|x+4|\cdot|x-3|<|x+4|\delta<(7+\delta)\delta.$$

Choose $\delta=\min\{1,\frac{\epsilon}{8}\}$, then $|(x^2+x-4)-8|<\epsilon$ if $|x-3|<\delta$.

2. a. Let $f(x)=\sin x-2+x$. Since $\sin x$ and polynomial $-2+x$ are continuous on $(-\infty,\infty)$, then $f(x)$ is continuous on $(-\infty,\infty)$. So, $f(x)$ is a continuous function.

   b. $f(0)=-2<0$ and $f(\pi)=-2+\pi>0 \Rightarrow$ By **I.V.T**, $\exists$ a root of $f(x)$ in $(0,\pi)$.

3. $\because -1\le\sin\frac{pi}{x}\le 1$ for $\forall x\in\mathbb{R}$, and $\sqrt{x^3+x^2}\ge 0$ for $x\in(-1,1)$
$\Rightarrow -\sqrt{x^3+x^2}\le\sqrt{x^3+x^2}\sin\frac{\pi}{x}\le\sqrt{x^3+x^2}$ for $-1<x<1$

And, $\lim\limits_{x\to 0}\sqrt{x^3+x^2}=\lim\limits_{x\to 0}(-\sqrt{x^3+x^2})=0\Rightarrow\lim\limits_{x\to 0}\sqrt{x^3+x^2}\sin\dfrac{\pi}{x}=0$ by the squeeze theorem.

4. 1^o It is clear that, for any real number a,b, $f(x)$ is continuous on the intervals that $x\neq 2,3$.

   2^o Assume that, $f(x)$ is also continuous at $x=2,3$. Then,

$$\begin{cases}\lim\limits_{x\to 2^-}f(x)=\lim\limits_{x\to 2^+}f(x)=f(2)\\ \lim\limits_{x\to 3^-}f(x)=\lim\limits_{x\to 3^+}f(x)=f(3)\end{cases}\Rightarrow\begin{cases}4a-2b+3=4\\ 9a-3b+3=6-a+b\end{cases}\Rightarrow\begin{cases}a=\frac{1}{2}\\ b=\frac{1}{2}\end{cases}$$

5. For $-\frac{1}{2}<x<\frac{1}{2}$, $|2x-1|=-(2x-1)$ and $|2x+1|=2x+1$.

$$(\text{for } -\frac{1}{2}<x<\frac{1}{2})\Rightarrow\lim_{x\to 0^+}\frac{|2x-1|-|2x+1|}{x}=\lim_{x\to 0^+}\frac{-(2x-1)-(2x+1)}{x}=\lim_{x\to 0^+}\frac{-4x+0}{x}=-4$$

$$\lim_{x\to 0^-}\frac{|2x-1|-|2x+1|}{x}=\lim_{x\to 0^-}\frac{-(2x-1)-(2x+1)}{x}=\lim_{x\to 0^-}\frac{-4x+0}{x}=-4$$

$$\Rightarrow\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}=-4$$