先修班　段考二　　**Jul 29, 2010**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘前不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。(請依題號順序依序寫在答案卷第一頁上)

1. If f is continuous at a, then f is differentiable at a.

2. If $g(x) = x^{1000}$, then $\lim_{x\to 1} \frac{g(x)-g(1)}{x-1} = 1000$.

3. If f has an absolute minimum value at c, then $f'(c) = 0$.

4. If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

5. If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$.

6. If f has a local maximum or minimum at c. Then c is a critical number of f.

7. If f is an even function, then f' is an even function.

8. $\frac{d}{dx}(tan^2 x) = \frac{d}{dx}(sec^2 x)$.

9. If f is increasing and $f(x) > 0$ on I, then $g(x) = \frac{1}{f(x)}$ is decreasing on I.

10. If f and g are both continuous, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

(下頁還有試題)

填充題 (*40* points)，每題 5 分。(請依題號順序依序寫在答案卷第一頁上)

1. Suppose the derivative of a function f is $f'(x) = (x+1)^2(x-3)^5(x-6)^4$. On what interval is f increasing?

Answer ： ___________.

2. Find the derivative of the following function:

$$y = \cos^4(\sin^3 x)$$

Answer ： ___________.

3. Determine whether $f'(0)$ exists.

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{if } x \neq 0, \\ 0 \quad , & \text{if } x = 0, \end{cases}$$

Answer ： ___________.

4. Find the derivative of the function: $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3$.

Answer ： ___________.

5. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possible be?

Answer ： ___________.

6. Find the local maximum and minimum values of $f(x) = x^4 - 2x^2 + 3$.

Answer ： ___________.

7. Evaluate dy of $y = \sqrt{4+5x}$ for the given values of $x = 0$ and $dx = 0.04$.

Answer ： ___________.

8. Find the derivative of $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$. Answer ： ___________.

(下頁還有試題)

計算問答證明題(*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

2. (*10* points) Use implicit differentiation to find an equation of the tangent line to the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$.

3. (*10* points) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$.

4. (*10* points) The figures shows a circular arc of length s and a chord of length d, both subtended by a central angle θ. Find $\lim_{\theta \to 0+} \frac{s}{d}$.

5. (*10* points) Find the absolute maximam and absolute minimum values of

$$f(x) = \frac{x^2 - 4}{x^2 + 4}$$

on the interval $[-4, 4]$.

(試題結束)

99學年暑期微積分先修班第二次段考解答

1 是非題

1.F 2.T 3.F 4.T 5.T 6.T 7.F 8.T 9.T 10.F

2 填充題

1. $x > 3$ 2. $-12\cos^3(\sin^3 x)\sin(\sin^3 x)\sin^2 x\cos x$ 3. 不存在 4. $-12\frac{x(x^2+1)^2}{(x^2-1)^4}$ 5. 16 6. local max:3 , local min:2 7. 0.05 8. $\frac{1}{2}\left(x+x\sqrt{x+\sqrt{x}}\right)^{-\frac{1}{2}}\left[1+\frac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}}(1+\frac{1}{2}x^{-\frac{1}{2}})\right]$

3 計算證明題

1. Let $f(x) = x^3 - 15x + c$, then $f'(x) = 3x^2 - 15 = 3(x+\sqrt{5})(x-\sqrt{5})$.
   $\Rightarrow f'(x) < 0$ on the interval $[-2, 2]$, so $f(x)$ is strictly decreasing on $[-2, 2]$,
   (**i.e.** $f(x_1) < f(x_2)$ if $x_1 > x_2$)

   Hence, if we suppose there exists a number d such that $f(d) = 0$, then $f(x) \neq 0$ if $x \neq d$.
   $\Rightarrow \exists$ no other root of $f(x)$.

2. $2(x^2+y^2)^2 = 25(x^2-y^2) \Rightarrow 4(x^2+y^2)(2x+2y\frac{dy}{dx}) = 50(x-y\frac{dy}{dx})$
   $\Rightarrow \frac{dy}{dx} = \frac{50x-8x(x^2+y^2)}{8y(x^2+y^2)+50y} = \frac{25x-4x(x^2+y^2)}{4y(x^2+y^2)+25y} \quad \therefore \left.\frac{dy}{dx}\right|_{(3,1)} = -\frac{9}{13}$
   Tangent line at the point $(3, 1)$: $y = -\frac{9}{13}x + \frac{40}{13}$.

3. $f'(x) = -\frac{1}{2\sqrt{1-x}}$, $f'(0) = -0.5$
   $\sqrt{0.9} = f(0.1) \approx f(0) + f'(0)(0.1-0) = 1 - 0.05 = 0.95$
   $\sqrt{0.99} = f(0.01) \approx f(0) + f'(0)(0.01-0) = 1 - 0.005 = 0.995$

4. 設半徑為r, 則 $s = r\theta$, $d = 2r\sin\frac{\theta}{2} \Rightarrow \lim_{\theta\to 0^+}\frac{s}{d} = \lim_{\theta\to 0^+}\frac{r\theta}{2r\sin\frac{\theta}{2}} = \lim_{\theta\to 0^+}\frac{\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 1$

5. $f(x) = \frac{x^2-4}{x^2+4}$, $f'(x) = \frac{16x}{(x^2+4)^2}$. $\Rightarrow f'(x) > 0$ for $x > 0$, $f'(0) = 0$, and $f'(x) < 0$ for $x < 0$.
   absolute minimum : $f(0) = -1$
   absolute maximum : $f(4) = f(-4) = \frac{3}{5}$