微積分聯合教學　段考四　**Mar 29, 2011**

考試時間 120 分鐘，題目卷爲三張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The point $\left(2, \frac{\pi}{2}\right)$ lies on the curve $r = 2\cos 2\theta$.

2. If both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent absolutely, then both $\sum_{n=1}^{\infty} (a_n + b_n)$ and $\sum_{n=1}^{\infty} (a_n - b_n)$ converge absolutely.

3. $\int_0^{\pi} \sqrt{1 + (\sin\theta)^2}\, d\theta = \int_0^{\pi} \sqrt{1 + (\cos\theta)^2}\, d\theta.$

4. If $\{a_n\}$ is an increasing and bounded above sequence then $\{a_n\}$ is convergent.

5. $\int_1^{\infty} \frac{1}{x^2}\, dx \le \sum_{n=2}^{\infty} \frac{1}{n^2}.$

6. $\frac{1}{2\sqrt{2}-1} + \frac{1}{3\sqrt{3}-1} + \frac{1}{4\sqrt{4}-1} + \frac{1}{5\sqrt{5}-1} + \ldots$ is divergent.

7. The series $\sum_{n=1}^{\infty} \left(\sqrt[n]{3} - 1\right)^n$ is convergent.

8. Assunme $a_n \neq 0$, $\forall n \in N$. If the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then the series $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right) a_n$ is also absolutely convergent.

(下頁還有試題)

9. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 10, then the raidus of convergence of the power series $\sum_{n=2}^{\infty} n(n-1)c_n x^{2n}$ is also 10.

10. Suppose $a_n > 0$ and $b_n > 0$ for all n, and there exist $m > 0$ and $M > 0$ such that $m \le \frac{b_n}{a_n} \le M$ for n large enough. Then $\sum a_n$ converges $\Longleftrightarrow$ $\sum b_n$ converges.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit of the sequence $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, ...\}$.

Answer ：___________.

2. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

Answer ：___________.

3. Find the arc length function for the curve $y = x^2 - \frac{1}{8}\ln x$ taking $P_0(1,1)$ as the starting point. Answer ：___________.

4. Find the slope of the tangent line to the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$ at the point where $\theta = \frac{\pi}{6}$.

Answer ：___________.

5. For what values of p is the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\ln n)^p}{n}$ convergent?

Answer ：___________.

6. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$.

Answer ：___________.

7. Find the Cartesian equation for the curve $r = 4\cos\theta$.

Answer ：___________.

8. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n(n+2)(n+4)}$. Answer ：___________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the sum of the series $\sum_{n=2}^{\infty} \frac{n^2 - n}{2^n}$.
(Hint: Use term-by-term differentiation.)

2. (*10* points) A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - 3t$.

a. Find the equation of the tangent line at $t = 2$.

b. Find $\frac{d^2y}{dx^2}$.

3. (*10* points)

a. Sketch the curve with polar equation $r = 1 + \sin\theta$.

b. Find the area of the region enclosed by the curve of the equation $r = 1 + \sin\theta$.

4. (*10* points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

a. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

b. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

5. (*10* points) Find the area of the surface obtained by rotating the circle

$$x^2 + (y - r)^2 = r^2$$

about the x-axis. ($r > 0$ is a constant.)

6. (*10* points) Find all positive values of b such that the series $\sum_{n=1}^{\infty} b^{\sqrt{n}}$ converges.
(Hint: Try Integral Test.)

(試題結束)