微積分聯合教學　段考五　**May 10, 2011**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名；題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材(包含手機)，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal, then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.

2. For any vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}\times\mathbf{v}=\mathbf{v}\times\mathbf{u}$ is true.

3. If $f_x(a,b)$ and $f_y(a,b)$ both exist, then f is differentiable at (a,b).

4. Suppose $\mathbf{a},\mathbf{b}\neq\mathbf{0}$. If $\mathbf{a}=-\mathbf{b}$, then $\text{proj}_{\mathbf{a}}\mathbf{b}=-\text{proj}_{\mathbf{b}}\mathbf{a}$

5. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\cdot(\mathbf{b}\times\mathbf{c})=0$

6. The equation $x^2-3y^2+9z^2+2x+5=0$ reprsents a hyperboloid of two sheets.

7. Let $\mathbf{r}(t)=\langle f(t),g(t),h(t)\rangle$ be a smooth vector function. If $|\mathbf{r}(t)|=1$ for all t, then $|\mathbf{r}(t)\times\mathbf{r}'(t)|=|\mathbf{r}'(t)|$.

8. If the nth derivative of $f(x)$ exists at $x=0$ for all n, then $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$.

(下頁還有試題)

9. Suppose that $\mathbf{a} \neq \mathbf{0}$. If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

10. There exists a function f with continuous second-order partial derivatives such that $f_x(x, y) = x + y^2$ and $f_y(x, y) = x - y^2$.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the area of the triangle with vertices $P(1, 4, 6)$, $Q(-2, 5, -1)$ and $R(1, -1, 1)$. Answer ：____________.

2. Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.

Answer ：____________.

3. If $\mathbf{a} = \langle 3, 0, -1 \rangle$, $\mathbf{b} = \langle 1, 2, z \rangle$, and the scalar projection $\text{comp}_{\mathbf{a}}\mathbf{b} = 2$, then evaluate z. Answer ：____________.

4. Let $f(x, y) = \begin{cases} \dfrac{2x^3 + y^3}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$.

Find $f_x(0, 0)$.

Answer ：____________.

5. Find the first three nonzero terms in the Maclaurin series for

$$f(x) = \int_0^x e^{-t^2}\, dt.$$

Answer ：____________.

6. Find the position vector of a particle that has acceleration

$$\mathbf{a}(t) = \langle 2\cos 2t, -2\sin 2t, 0 \rangle$$

with initial velocity $\mathbf{v}(0) = \langle 2, 1, 0 \rangle$ and initial position $\mathbf{r}(0) = \langle 3, 2, 1 \rangle$.

Answer ：____________.

7. Find the linearization of $f(x, y) = xe^{xy}$ at $(1, 0)$. Answer ：____________.

8. Find $\dfrac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1$. Answer ：____________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find equations of (**a.**) the tangent plane (**b.**) the normal line to the surface $z + 1 = xe^y \cos z$ at the point $(1, 0, 0)$.

2. (*10* points) Use a scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.

3. (*10* points)

a. Reparametrize

$$\mathbf{r}(t) = e^{2t} \cos 2t \, \mathbf{i} + 2 \, \mathbf{j} + e^{2t} \sin 2t \, \mathbf{k}$$

with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

b. Find the vectors **T**, **N**, and **B** of the curve $\mathbf{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$ at the given point $\left(1, \frac{2}{3}, 1\right)$.

4. (*10* points) Find the maximum rate of change of $f(x, y) = \sin(xy)$ at $(1, 0)$ and the direction in which it occurs.

5. (*10* points) Determine the set of points at which the function f is continuous. Please justify your answer.

$$f(x, y) = \begin{cases} \dfrac{x^3 y^2}{x^5 + 2y^5} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases} .$$

(試題結束)