微積分聯合教學　段考六　**Jun 21, 2011**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名；題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材(包含手機)，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\{a_n\}$ is an increasing and bounded above sequence then $\{a_n\}$ is convergent.

2. For any vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}\times\mathbf{v}=\mathbf{v}\times\mathbf{u}$ is true.

3. There exists a function f with continuous second-order partial derivatives such that $f_x(x,y)=x+y^2$ and $f_y(x,y)=x-y^2$.

4. Let f be a continuous function defined on a smooth plane curve C given by the parametric equations $x=x(t)$, $y=y(t)$, $a\le t\le b$. If $-C$ denotes the curve consisting of the same points as C but with the opposite orientation, then we have $\int_{-C} f(x,y)\,dx=-\int_C f(x,y)\,dx$ and $\int_{-C} f(x,y)\,ds=\int_C f(x,y)\,ds$.

5. $\int_0^1\int_0^x \sqrt{x+y^2}\,dydx=\int_0^x\int_0^1\sqrt{x+y^2}\,dxdy$.

6. If D is the disk given by $x^2+y^2\le 4$, then $\int\int_D \sqrt{4-x^2-y^2}\,dA=\frac{16}{3}\pi$.

7. If $\mathbf{F}$ is a vector field, then div $\mathbf{F}$ is a vector field.

8. If f has continuous partial derivatives on $\mathbb{R}^3$ and C is any circle, then

$$\int_C \nabla f\cdot d\mathbf{r}=0.$$

9. The vector field $\mathbf{F}(x,y)=(x-y)\mathbf{i}+(x-2)\mathbf{j}$ is conservative.

10. There is a vector field $\mathbf{F}$ such that curl$\mathbf{F}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the Cartesian equation for the curve $r = 4\cos\theta$.

Answer ：__________.

2. Let $f(x,y) = \begin{cases} \dfrac{2x^3+y^3}{x^2+y^2} & ,\ (x,y) \neq (0,0) \\ 0 & ,\ (x,y) = (0,0) \end{cases}$.

Find $f_x(0,0)$.

Answer ：__________.

3. Evaluate the line integral $\int_C (2x+9z)ds$, where the curve C is given by $x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 1$.

Answer ：__________.

4. The density at any point on a semicircular lamina D (薄板) is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

Answer：__________.

5. Find the saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

Answer ：__________.

6. Convert the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\,dzdydx$ to an equivalent integral in cylindrical coordinates. (**Do not evaluate the integral**).

Answer ：__________.

7. If $\mathbf{F}(x,y,z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - y^2\,\mathbf{k}$, find div $\mathbf{F}$.

Answer ：__________.

8. Evaluate $\oint_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4+1})\,dy$, where C is the circle $x^2+y^2 = 9$.

Answer ：__________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

2. (*10* points) Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2)\, dydx$.

3. (*10* points) Evaluate the integral $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$, by making an appropriate change of variables.

4. (*10* points) Find the work done by the force field $\mathbf{F}(x, y) = x\mathbf{i} + (y + 2)\mathbf{j}$ in moving an object along an arch of the cycloid $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$, $0 \le t \le 2\pi$.

5. (*10* points)

a. Show that $\mathbf{F}(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k}$ is a conservative vector field.

b. Find a function f such that $\mathbf{F} = \nabla f$.

(試題結束)