CALCULUS(I)–FIRST MIDDLE EXAMINATION

1. (10pts) Show that (using "$\epsilon - \delta$") if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$, then $\lim_{x\to a}(f(x) + g(x)) = L + K$.

2. (24pts)
   (a) (6pts) Find $\lim_{x\to 1} \dfrac{x^2 - 1}{\sqrt{2x+2} - 2}$.
   (b) (6pts) Find $\lim_{x\to 0} \dfrac{3x}{\tan 5x}$.
   (c) (6pts) Find $\dfrac{d^2}{dx^2}\left((x^2 - 3x)\dfrac{d}{dx}(x^{-1})\right)$.
   (d) (6pts) Find the tangent and normal lines of the equation $\tan(xy) = x$ at the point $(1, \pi/4)$.

3. (10pts) Find the absolute extrema of $f(x) = 3x^{\frac{2}{3}} - 2x$ on $[-1, 2]$.

4. (10pts) A power line is needed to connect a power station on the shore of a river to an island 4 kilometers downstream and 1 kilometer offshore. Find the minimum cost for such a line given that it costs $\$50,000$ per kilometer to lay wire under water and $\$40,000$ per kilometer to lay wire under ground.

5. Sketch the graph of the following functions. Label the intercepts, relative extrema, points of inflection, and asymptotes.
   a. (13pts) $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$.
   b. (13pts) $f(x) = \dfrac{x^2 - 6x + 12}{x - 4}$.

6. (10pts) Let $f(x)$ be continuous and $0 \le f(x) \le 1$ on $[0, 1]$. Also let f be differentiable and $f'(x) < 1$ on $(0, 1)$. Show that $f(x)$ has exactly one fixed point in $[0, 1]$.

7. (10pts) Newton's Method.
   (a) (5pts) Draw a picture and deduce from it the Newton's iteration process from x_n to x_{n+1} to approximate roots of $f(x) = 0$.
   (b) (5pts) Show that for $f(x) = 2x^3 - 3x^2 - 1$, there is exactly one root $c \in (1, 2)$ for $f(x) = 0$. Let $x_1 = 2$, compute x_2, x_3.