CALCULUS(I)–SECOND MIDDLE EXAMINATION

1. (10pts) Let $f(x)$ be continuous on $[a, b]$ and $F(x) := \int_a^x f(t)dt$. Show that $F(x)$ is continuous on $[a, b]$, differentiable on (a, b) and $F'(x) = f(x)\ \forall x \in (a, b)$.

2. (20pts)
   (a) Using Riemann Sum to evaluate $\int_0^1 x^2 dx$.
   (b) Evaluate the following limit by expressing the limit as a definite integral on $[0, 1]$:
   $$\lim_{n\to\infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{n}{n^2+n^2} \right).$$

3. (10pts)
   (a) Let $F(x) = \displaystyle\int_1^x \log_5(3t^2 + 7t + 1)dt$. Determine $F''(x)$.
   (b) Find a function f that satisfies the conditions : $f''(x) = x^{-2/3}$, $f'(8) = 6$, $f(0) = 0$.

4. (10pts) Find the area of the region bounded by $y = x^3 - 4x^2 + 1$ and $y = x - 3$.

5. (10pts) The circular disc $x^2+y^2 \le 4$ is revolved about the line $x = 2$. Find the volume of the solid that is generated.

6. (36pts) Compute indefinite integrals:
   (a) $\displaystyle\int x \sin^3 x^2 \cos x^2\, dx$; (b) $\displaystyle\int \csc x\, dx$;
   (c) $\displaystyle\int x(\ln x)^2\, dx$; (d) $\displaystyle\int \frac{x^2}{1+x^6}\, dx$;
   (e) $\displaystyle\int \frac{3x^3 + 11x^2 + 23x + 6}{x^3 + 2x^2 + x}\, dx$; (f) $\displaystyle\int (2^x + x)^2\, dx$.

7. (10pts)
   (a) Differentiate $f(x) = (\sin x)^x$.
   (b) Differentiate $F(x) = \displaystyle\int_{x^3}^3 \frac{\sin t}{t}\, dt$

8. (10pts) Let Ω be the annular region formed by the circles $x^2+y^2 = \frac{1}{4}$, $x^2 + y^2 = 4$. Locate the centroid of the first-quadrant part of Ω.