CALCULUS(I)– FINAL EXAMINATION

1. (40pts) Compute indefinite integrals:
   (a) $\int \frac{1}{x}\sin^{-1}(\ln x)\,dx$; (b) $\int \frac{x}{\sqrt{x^2+4x+3}}\,dx$;
   (c) $\int \tan^5 x \sec^5 x\,dx$; (d) $\int \sin^5 x\,dx$;
   (e) $\int (e^x + x^2)^2\,dx$.

2. (10pts) Approximate $\ln 2 = \int_1^2 \frac{1}{x}dx$ using Simpson's Rule with $n = 5$. Find a bound on the error for the approximation. (Note: the error formula is given by $\frac{(b-a)^5}{2880n^4}[\max_{x\in[a,b]} |f^{(4)}(x)|]$.)

3. (10pts) Evaluate (a) $\int_1^\infty \frac{1}{x^p}dx$, (b) $\int_0^2 \frac{1}{(x-1)^{2/3}}dx$.

4. (10pts) Find (a) $\lim_{x\to\infty} \frac{(\ln x)^3}{x}$, (b) $\lim_{x\to 1^+} (x-1)^{(x-1)}$.

5. (30pts)Determine whether each series converges or diverges. Justify your answer.
   (a) $\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$ (b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$
   (c) $\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n}$ (d) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$
   (e) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ (f) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n-1)}{n^3+1}$

6. (20pts)
   (a) Apply Taylor's Theorem to find the power series centered at 3 for $f(x) = e^{2x}$ and find the radius of the convergence.
   (b) Use Taylor polynomials to estimate $e^{0.2}$ within 0.01.
   (c) Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{n}{2^n}(x-1)^n$.