CALCULUS(II)– FINAL EXAMINATION

1. (10%) Use spherical coordinates to evaluate
$$\iiint_T z\sqrt{x^2+y^2+z^2}\,dxdydz$$
where $T: 0 \le x \le \sqrt{9-y^2},\ 0 \le y \le 3,\ 0 \le z \le \sqrt{9-x^2-y^2}$.

2. (10%) Find the volume of T which is bounded by $x+y+z=0$, $x+y+z=1,\ x-y=-1,\ x-y=2,\ x+y-z=1,\ x+y-z=3$.

3. (10%) Calculate the work done by the force
$$F(x,y,z) = 3x^2\mathbf{i} + \frac{z^2}{y}\mathbf{j} + 2z\ln y\,\mathbf{k}$$
applied to a particle that moves from $(0,1,1)$ to $(2,2,1)$.

4. (10%) Evaluate $\int_C x^2y\,dx + y\,dy + xz\,dz$ where C is the curve of intersection of $y-2z^2=1$ and $z=x+1$ from $(0,3,1)$ to $(1,9,2)$.

5. (10%) Evaluate $\int_C [2\tan^{-1}\frac{y}{x}\,dx + \ln(x^2+y^2)\,dy]$, where C is the circle $(x-2)^2+y^2=1$ traversed in a counterclockwise manner.

6. (10%) Calculate the area of the surface $4-(x^2+y^2)$ with $1 \le x^2+y^2 \le 4$.

7. (10pts) Evaluate
$$\iint_S (\mathbf{v}\cdot\mathbf{n})\,d\sigma$$
where $\mathbf{v} = y\mathbf{i} - x\mathbf{j} - 4z^2\mathbf{k}$ and S is that portion of the cone $z = \sqrt{x^2+y^2}$ that lies above the square in the xy-plane with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$. Take $\mathbf{n}$ as the downward unit normal.

8. (10%) Calculate the flux of $\mathbf{v}(x,y,z) = [(2x-1)(x^2+y^2+z^2)+2xy]\mathbf{i}+[(2y-1)(x^2+y^2+z^2)+2yz]\mathbf{j}+[(2z-1)(x^2+y^2+z^2)+2xz]\mathbf{k}$ out of the sphere $S: x^2+y^2+z^2=4$.

9. (10%) Is $\mathbf{v}(x, y, z) = (2xz + \sin y)\mathbf{i} + x \cos y\mathbf{j} + x^2\mathbf{k}$ a gradient? Evaluate the line integral of $\mathbf{v}(x, y, z)$ over the curve $C : r(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$, $t \in [0, 2\pi]$.

10. (10pts) The upper half of the ellipsoid $x^2 + y^2 + 2z^2 = 2$ intersects the cylinder $x^2 + y^2 - y = 0$ in a curve C. Calculate the circulation of $\mathbf{v}(x, y, z) = y^3\mathbf{i} + (xy + 3xy^2)\mathbf{j} + z^4\mathbf{k}$ around C in the clockwise manner.