CALCULUS(II) – FINAL EXAMINATION

1. (10%) Use spherical coordinates to evaluate

\[ \iiint_T z \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz \]

where \( T : 0 \leq x \leq \sqrt{9-y^2}, 0 \leq y \leq 3, 0 \leq z \leq \sqrt{9-x^2-y^2}. \)

2. (10%) Find the volume of \( T \) which is bounded by \( x + y + z = 0, \) \( x + y + z = 1, \) \( x - y = -1, \) \( x + y - z = 2, \) \( x + y - z = 1, \) \( x + y - z = 3. \)

3. (10%) Calculate the work done by the force

\[ F(x, y, z) = 3x^2 \mathbf{i} + \frac{z^2}{y} \mathbf{j} + 2z \ln y \mathbf{k} \]

applied to a particle that moves from \((0, 1, 1)\) to \((2, 2, 1)\).

4. (10%) Evaluate \( \int_C x^2 y \, dx + y \, dy + xz \, dz \) where \( C \) is the curve of intersection of \( y - 2z^2 = 1 \) and \( z = x + 1 \) from \((0, 3, 1)\) to \((1, 9, 2)\).

5. (10%) Evaluate \( \int_C [2 \tan^{-1} \frac{y}{x} \, dx + \ln(x^2 + y^2) \, dy], \) where \( C \) is the circle \((x - 2)^2 + y^2 = 1\) traversed in a counterclockwise manner.

6. (10%) Calculate the area of the surface \( 4 - (x^2 + y^2) \) with \( 1 \leq x^2 + y^2 \leq 4. \)

7. (10pts) Evaluate

\[ \iint_S (\mathbf{v} \cdot \mathbf{n}) \, d\sigma \]

where \( \mathbf{v} = y \mathbf{i} - x \mathbf{j} - 4z^2 \mathbf{k} \) and \( S \) is that portion of the cone \( z = \sqrt{x^2 + y^2} \) that lies above the square in the \( xy \)-plane with vertices \((0, 0), (1, 0), (1, 1), \) and \((0, 1)\). Take \( \mathbf{n} \) as the downward unit normal.

8. (10%) Calculate the flux of \( \mathbf{v}(x, y, z) = [(2x - 1)(x^2 + y^2 + z^2) + 2xy] \mathbf{i} + [(2y - 1)(x^2 + y^2 + z^2) + 2yz] \mathbf{j} + [(2z - 1)(x^2 + y^2 + z^2) + 2xz] \mathbf{k} \)
out of the sphere \( S : x^2 + y^2 + z^2 = 4. \)
9. (10%) Is \( \mathbf{v}(x, y, z) = (2xz + \sin y)i + x \cos yj + x^2k \) a gradient? Evaluate the line integral of \( \mathbf{v}(x, y, z) \) over the curve \( C : r(t) = \cos t i + \sin t j + tk, \; t \in [0, 2\pi] \).

10. (10pts) The upper half of the ellipsoid \( x^2 + y^2 + 2z^2 = 2 \) intersects the cylinder \( x^2 + y^2 - y = 0 \) in a curve \( C \). Calculate the circulation of \( \mathbf{v}(x, y, z) = y^3i + (xy + 3xy^2)j + z^4k \) around \( C \) in the clockwise manner.