**1.** Evaluate the limits that exist:

$$\text{(a)} \lim_{x\to -1} \frac{x^2+1}{3x^5+4} \qquad \text{(b)} \lim_{x\to 0} x\left(\frac{1}{x}-1\right)$$

**2.** Give an $\epsilon$-$\delta$ proof for the limit $\lim_{x\to 2} |x-4| = 2$.

**3.** Let

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Prove that $\lim_{x\to 0} f(x)$ does not exist.

**4.** Solve the inequality $x(2x-1)(3x-5) \leq 0$ for $x$.

**5.** For $x \in (0,10) \setminus \{5\}$, let $f(x) = \dfrac{\sqrt{x+4}-3}{x-5}$. Define the function $f$ at $x=5$ so that $f$ becomes continuous at 5.

**6.** Prove that if there is a number $A$ such that $\left|\dfrac{g(x)}{x-1}\right| \leq A$ for all $x \neq 1$, then $\lim_{x\to 1} g(x) = 0$.

**7.** Let $f(x) = \dfrac{1}{x+1} + \dfrac{1}{x+4}$. Show that there is a number $c \in (-4,-1)$ such that $f(c) = 0$.

**8.** Find numbers $A$ and $B$ such that the function

$$f(x) = \begin{cases} x^3, & \text{if } x \leq 1, \\ Ax+B, & \text{if } x > 1. \end{cases}$$

is differentiable at $x=1$.