1. Evaluate the limits that exist:

\[(a) \lim_{x \to -1} \frac{x^2 + 1}{3x^5 + 4} \quad (b) \lim_{x \to 0} x\left(\frac{1}{x} - 1\right)\]

2. Give an \( \varepsilon-\delta \) proof for the limit \( \lim_{x \to 2} |x - 4| = 2 \).

3. Let

\[f(x) = \begin{cases} 
1, & \text{if } x \text{ is rational} \\
0, & \text{if } x \text{ is irrational.}
\end{cases}\]

Prove that \( \lim_{x \to 0} f(x) \) does not exist.

4. Solve the inequality \( x(2x - 1)(3x - 5) \leq 0 \) for \( x \).

5. For \( x \in (0, 10) \setminus \{5\} \), let \( f(x) = \frac{\sqrt{x + 4} - 3}{x - 5} \). Define the function \( f \) at \( x = 5 \) so that \( f \) becomes continuous at \( 5 \).

6. Prove that if there is a number \( A \) such that \( \left| \frac{g(x)}{x - 1} \right| \leq A \) for all \( x \neq 1 \), then \( \lim_{x \to 1} g(x) = 0 \).

7. Let \( f(x) = \frac{1}{x + 1} + \frac{1}{x + 4} \). Show that there is a number \( c \in (-4, -1) \) such that \( f(c) = 0 \).

8. Find numbers \( A \) and \( B \) such that the function

\[f(x) = \begin{cases} 
x^3, & \text{if } x \leq 1, \\
Ax + B, & \text{if } x > 1.
\end{cases}\]

is differentiable at \( x = 1 \).