

1. (a) For $x > 1$, find $\frac{d}{dx}[(\ln x)^{\ln x}]$.
(b) Evaluate $\int_1^e \frac{\ln u}{u} du$.
(c) Evaluate $\int_0^4 f(x) dx$, where $f(x) = \begin{cases} 2x + 1, & 0 \leq x \leq 1 \\ 3e^{x-1}, & 2 < x \leq 4. \end{cases}$
2. If function f is one-to-one and continuous on open interval (a, b) , then f either increases throughout (a, b) or it decreases throughout (a, b) .
3. Prove or disprove that the range of $y = \ln x$ is $(-\infty, \infty)$.
4. For $x \in \mathbb{R}$, prove $\frac{d}{dx}(e^x) = e^x$.
5. Prove that $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \forall x \in \mathbb{R}$.
6. Prove that $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \forall x \in (-1, 1)$.
7. For $x, y \in \mathbb{R}$, prove that $\sinh(x+y) = \sinh(x) \cosh(y) + \cosh(x) \sinh(y)$.
8. Prove that $f(x) > 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} \forall x > 0$ & $n \in \mathbb{N}$, where $f(x)$ is the inverse function of $\ln x$.