

- (a) For $\alpha > 0$, use a trigonometric substitution to show that $\int \frac{1}{\sqrt{\alpha^2 + x^2}} dx = \ln |x + \sqrt{\alpha^2 + x^2}| + C$.
(b) Let $m \neq n$ be positive integers. Calculate the integral $\int (\sin mx)(\sin nx) dx$.
- Suppose that the real numbers α and β are not both zero, then identify the curve $r = \alpha \cos \theta + \beta \sin \theta$ and find the equation in rectangular coordinates for it.
- Sketch the curves $r = 1 - \cos \theta$ and $r = 1 + \sin \theta$ and find the points at which they intersect.
- Find the area outside $r = \cos 2\theta$, but inside $r = 1$.
- Find the tangent(s) to the curve $x(t) = t \sin \frac{\pi t}{2}$, $y(t) = t^3 - t$ at the point $(1, 0)$.
- Determine whether the tangent line at $[0, 0]$ of the curve $r = 1 - \cos \theta$, $\theta \in [-\pi, \pi]$ exist or not?
- Write the polar equation $r^2 = \sin \theta$ in rectangular coordinates.
- Let $x = x(t)$ be continuous on $[c, d]$ with $x(t) \in [a, b]$ for $t \in [c, d]$, $x(c) = a$ and $x(d) = b$. Let $y = f(x)$ be a function on $[a, b]$. If $y = f(x) = f(x(t))$, as a function of t , is continuous on $[c, d]$, then $y = f(x)$, as a function of x , is continuous on $[a, b]$.