

1. Find the sum of the series $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)}$.
2. Suppose that for all positive integer n greater than some inter M , the sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ satisfy $a_n \leq b_n \leq c_n$. If $\exists L \in R$ such that $a_n \rightarrow L$ as $n \rightarrow \infty$ and $c_n \rightarrow L$ as $n \rightarrow \infty$, prove that $b_n \rightarrow L$ as $n \rightarrow \infty$.
3. Prove that the series $\sum_{m=1}^{\infty} \frac{1}{m \ln(m+1)}$ diverges.
4. Let α and β be positive numbers. Prove or disprove that $\sqrt{\alpha\beta} = \lim_{x \rightarrow \infty} \left(\frac{\alpha^{\frac{1}{x}} + \beta^{\frac{1}{x}}}{2} \right)^x$.
5. Find $\lim_{x \rightarrow 0} \frac{e^{-1/x^2}}{x}$.
6. Evaluate $\int_1^4 \frac{dx}{x^2-4}$.
7. Find the values of p for which the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ diverges.
8. Let F be a differentiable vector-valued function. Prove or disprove that if $\|F(t)\| \neq 0$, then $\frac{d}{dt}(\|F(t)\|) = \frac{F(t) \cdot F'(t)}{\|F(t)\|}$.