1. Evaluate $\int_C (y^2+2x+1)dx + (2xy+4y-1)dy$ along the path $C: y = x^2$ from (0,0) to (1,1).
2. Show that the area of the region Ω enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.
3. Set $P(x,y)=\frac{y}{x^2+y^2}$ and $Q(x,y)=\frac{-x}{x^2+y^2}$ on the punctured unit disc $\Omega: 0 < x^+y^2 < 1$.
   (a)Verify that P and Q are continuously differentiable on Ω and that $\frac{\partial P}{\partial y}(x,y)=\frac{\partial Q}{\partial x}(x,y)$ for all $(x,y)\in\Omega$.
   (b)Verify that the vector field $\vec{h}(x,y)=P(x,y)\vec{i}+Q(x,y)\vec{j}$ is not a gradient on Ω.
4. Find the directional derivative of $g(x,y)=\ln\sqrt{x^2+y^2}$ at (3,4) toward the origin.
5. Find a function f with the gradient $F(x,y,z)=yz\vec{i}+(xz+2yz)\vec{j}+(xy+y^2)\vec{k}$.
6. Let f be a real valued function of 3 variables which is defined in some neighborhood of $\overline{x}$. If there exist vectors $\overline{y_1}$ and $\overline{y_2}$ such that $[f(\overline{x}+h)-f(\overline{x})]-\overline{y_1}\cdot h=o(h)$ and $[f(\overline{x}+h)-f(\overline{x})]-\overline{y_2}\cdot h=o(h)$, then $\overline{y_1}=\overline{y_2}$.
7. Let $C: \vec{r}(u)=x(u)\vec{i}+y(u)\vec{j}+z(u)\vec{k}$, $u\in[a,b]$ be a smooth curve and $\vec{h}(x,y,z)=h_1(x,y,z)\vec{i}+h_2(x,y,z)\vec{j}+h_3(x,y,z)\vec{k}$ be continuous on C. Prove that $\int_{-C}\vec{h}(\vec{R})\cdot d\vec{R}=-\int_C\vec{h}(\vec{r})\cdot d\vec{r}$, where curve $-C:\vec{R}(w)=\vec{r}(a+b-w)$, $w\in[a,b]$.
8. Evaluate $\oint_C x^2dy$ by *Green's theorem*, where C is the rectangle with vertices (0,0), (3,0), (3,4), (0,4).