

1. (a) (3 pts) Find  $\lim_{x \rightarrow -4} \frac{x^2 - 16}{x^2 + 5x + 4}$ .
- (b) (3 pts) Find  $\lim_{x \rightarrow 0} \frac{x^2 - 2x}{\sin 3x}$ . [Hint:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .]
- (c) (5 pts) Find  $f'(x)$  where  $f(x) = \frac{\sec \frac{1}{x}}{(2 - x^2)^{1/3}}$
- (d) (5 pts) Let  $F(x) = \int_0^{\cos^4 2x} \frac{1}{t^2 + 1} dt$ . Find  $F'(x)$ .
- (e) (4 pts) Express the following limit as a definite integral on the indicated interval.

$$\lim_{\|P\| \rightarrow 0} \left[ t_1^2 \tan(3t_1 - 1) \Delta t_1 + t_2^2 \tan(3t_2 - 1) \Delta t_2 + \cdots + t_n^2 \tan(3t_n - 1) \Delta t_n \right]$$

where  $t_i \in [t_i, t_i + 1]$ ,  $i = 1, \dots, n$ ;  $[1, 4]$ .

2. (6 pts) Let  $f(x) = \frac{1}{\sqrt{x}}$ . Compute  $f'(x)$  by using the *definition of the derivative* as a **limit**. (You will receive 0 point for this problem if you compute  $f'(x)$  by using other methods.)
3. (8 pts) Find the equation of the tangent line to the curve  $\sin(x + y) = (x + y)xy$  at the point  $(1, -1)$ .
4. An object moves along the  $x$ -axis, its position at time  $t$  is  $x(t) = t^3 - 6t^2 + 9t + 2$ .
  - (a) (4 pts) Find the time(s) when it changes direction.
  - (b) (4 pts) Find the time interval(s) on which the object is moving left and slowing down.
5. (8 pts) Find the area of the region bounded by the curves  $y = x + 1$  and  $y = x^2 - x + 1$ .
6. (15 pts) Let  $f(x) = 1 + \frac{2}{x} - \frac{1}{x^2}$ .
  - (a) Find all points where  $f$  is either undefined or not continuous.
  - (b) Find the intervals where  $f$  is increasing/decreasing. Find and classify all local extrema.
  - (c) Find the intervals where the graph of  $f$  is concave up/down. Find all inflection points.
  - (d) Find all horizontal and vertical asymptotes of the graph of  $f$ .
  - (e) Sketch the graph of  $f$ .
7. (10 pts) Water is dripping through the bottom of a conical cup 6 inches across and 6 inches deep. Given the top loses half a cubic inch of water per minute, how fast is the water level dropping when the water is 2 inches deep?
8. (10 pts) A rectangular box with square base and top is to be made to contain 1000 cubic feet. The material for the base costs 25 cents per square foot, for the top 15 cents per square foot, and for the sides 10 cents per square foot. Find the dimension that will minimize the cost of the box. You must justify your answer.
9. (5 pts) A box is to be constructed in the form of a cube to hold 1000 cubic feet. Use a differential to estimate how accurately the inner edge must be made so that the volume will be correct to within 1 cubic feet.
10. Let  $f(x) = x^9 + x - 1$ .
  - (a) (5 pts) Show that  $f$  must have a root on the interval  $[0, 1]$ .
  - (b) (5 pts) Show that  $f$  has *only one* real root.