

1. (10 pts) Let  $f(x) := \int_{\frac{1}{x}}^x \sqrt{1+t^2} dt$ ,  $x > 0$ .

(a) Find  $f'(x)$ . (b) Prove that  $f$  has an inverse. (c) Find  $(f^{-1})'(0)$ .

2. (5 pts) Find the derivative  $\frac{d}{dx}(\sinh x)^{(2^x)}$ .

3. (10 pts) Find the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{x - \tan x}{x - \sin x}$ .

(b)  $\lim_{n \rightarrow \infty} (\ln n)^{\frac{1}{n}}$ .

4. (32 pts) Compute indefinite integrals.

(a)  $\int x^2 e^x dx$ .

(b)  $\int \sin^{3/2} x \cos^3 x dx$ .

(c)  $\int \frac{x}{\sqrt{x^2 + 4x + 3}} dx$ .

(d)  $\int \frac{2x^3 + x^2 + 3x + 1}{x^3 + x} dx$ .

5. (5 pts) Evaluate the improper integral  $\int_0^2 \frac{1}{(x-1)^{2/3}} dx$ .

6. (16 pts) Let  $\Omega$  be the region bounded by the graph of  $y = \sin x$  between  $x = 0$  and  $x = \pi$ . Find the area of the solid obtained by revolving  $\Omega$  about (a) the  $x$ -axis, (b) the  $y$ -axis.

7. (20 pts) Determine whether each series converges or diverges. Justify your answer.

(a)  $\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$ .

(b)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ .

(c)  $\sum_{n=1}^{\infty} \left(\frac{3n^2+1}{n^2+n}\right)^n$ .

(d)  $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdots 2n}{(2n)!}$ .

8. (7 pts) Find the interval of convergence of  $\sum_{n=1}^{\infty} \frac{1}{n2^n}(x-1)^n$ .

9. (7 pts) Expand the function  $\ln x$  as a Taylor series in powers of  $(x-1)$ .

10. (8 pts) Let  $f(x) = e^{3x^3}$ . Do the following problems by using the identity  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ ,  $x \in \mathbb{R}$ .

(a) Expand  $f(x) = e^{3x^3}$  in powers of  $x$  and determine the value of  $f^{(6)}(0)$ .

(b) Let  $F(x)$  be the function satisfying  $F'(x) = f(x)$  and  $F(0) = 1$ . Expand  $F(x)$  in powers of  $x$ .