1. (6 pts) Let $\mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$. Find vectors $\mathbf{b}$ and $\mathbf{c}$ so that
$$\mathbf{u} = \mathbf{b} + \mathbf{c}, \quad \mathbf{b} \parallel \mathbf{v}, \quad \mathbf{c} \perp \mathbf{v}.$$

2. (12 pts) Find the distance between
   (a) two lines $l_1 : \mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ and $l_2 : \mathbf{R}(u) = u(\mathbf{i} + \mathbf{j} + \mathbf{k})$;
   (b) two planes $S_1 : x + 2y + 2z = 11$ and $S_2 : 3x + 6y + 6z = 5$.

3. (8 pts) Find the indicated derivative
$$\frac{d}{dt}\Big[(e^{-2t}\mathbf{j} - 5\mathbf{j} + \ln t\mathbf{k}) \times (\cos^2 t\mathbf{i} + t^3\mathbf{j} + \frac{1}{t}\mathbf{k})\Big].$$

4. (10 pts) Let $\mathbf{f}'(t) = t^2\mathbf{i} + t(1+t^2)^{-1}\mathbf{j} + te^t\mathbf{k}$ and $\mathbf{f}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Find $\mathbf{f}(t)$.

5. (24 pts) Consider a parametrized Helix $\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}$, $0 \le t \le 4\pi$.
   (a) Find an equation in x, y, z for the osculating plane of the curve at $t = \pi/2$.
   (b) Find the length of the curve and parametrize the curve by arc length.
   (c) Determine its curvature at any time t.

6. (6 pts) Consider the motion of a particle with position vector $\mathbf{r}(t)$ which is constraint by a central force $\mathbf{F}(\mathbf{r}) = f(\mathbf{r})\mathbf{r}$. Show that the angular momentum is a constant at any t.

7. (8+4 pts)
   (a) Let $f(x, y) = \dfrac{x}{x+y}$. Determine the domain of f and sketch its c-level sets for $c = -1, 0, 1, 2$.
   (b) Find an equation for the level surface of $g(x, y, z) = x^2 + 2y^2 - 2xyz$ that contains the point $(-1, 2, 1)$.

8. (16 pts) Identify the surfaces below and find the traces (intersections with the coordinate planes). Then sketch the surfaces.
$$(a)\ \ x^2 - y^2 - z = 0. \qquad\qquad (b)\ \ x^2 - y^2 - z^2 = -1.$$

9. (10 pts) Find the partial derivatives f_x and f_y (and f_z).
$$(a)\ \ f(x, y) = y\tan^{-1}(xy^2). \qquad\qquad (b)\ \ f(x, y, z) = \left(\frac{x}{y}\right)^z.$$

10. (16 pts) The surface $S : z = \sqrt{6 - x^2 - y^2}$ is a hemisphere of radius $\sqrt{6}$ centered at the origin.
    (a) Find a vector parametrization of the line l_1 tangent to the curve of intersection of S with the plane $x = 1$ at the point $(1, 2, 1)$.
    (b) Let C be the intersection of S with the plane $z + y = 2$. Determine the projection of C onto the xy-plane and find a vector parametrization for it.