1. (6 pts) Let \( \mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{v} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} \). Find vectors \( \mathbf{b} \) and \( \mathbf{c} \) so that 
\[
\mathbf{u} = \mathbf{b} + \mathbf{c}, \quad \mathbf{b} \parallel \mathbf{v}, \quad \mathbf{c} \perp \mathbf{v}.
\]

2. (12 pts) Find the distance between

(a) two lines \( l_1 : \mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \) and \( l_2 : \mathbf{R}(u) = u(\mathbf{i} + \mathbf{j} + \mathbf{k}); \)

(b) two planes \( S_1 : x + 2y + 2z = 11 \) and \( S_2 : 3x + 6y + 6z = 5. \)

3. (8 pts) Find the indicated derivative
\[
\frac{d}{dt} \left[ (e^{-2t}\mathbf{j} - 5\mathbf{j} + \ln t\mathbf{k}) \times (\cos 2t\mathbf{i} + t^3\mathbf{j} + 1/t\mathbf{k}) \right].
\]

4. (10 pts) Let \( f'(t) = t^2\mathbf{i} + t(1 + t^2)^{-1}\mathbf{j} + te^t\mathbf{k} \) and \( f(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}. \) Find \( f(t). \)

5. (24 pts) Consider a parametrized Helix \( \mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}, \ 0 \leq t \leq 4\pi. \)

(a) Find an equation in \( x, y, z \) for the osculating plane of the curve at \( t = \pi/2. \)

(b) Find the length of the curve and parametrize the curve by arc length.

(c) Determine its curvature at any time \( t. \)

6. (6 pts) Consider the motion of a particle with position vector \( \mathbf{r}(t) \) which is constrained by a central force \( \mathbf{F}(\mathbf{r}) = f(\mathbf{r})\mathbf{r}. \) Show that the angular momentum is a constant at any \( t. \)

7. (8+4 pts)

(a) Let \( f(x, y) = \frac{x}{x + y}. \) Determine the domain of \( f \) and sketch its \( c \)-level sets for \( c = -1, 0, 1, 2. \)

(b) Find an equation for the level surface of \( g(x, y, z) = x^2 + 2y^2 - 2xyz \) that contains the point \((-1, 2, 1). \)

8. (16 pts) Identify the surfaces below and find the traces (intersections with the coordinate planes). Then sketch the surfaces.

(a) \( x^2 - y^2 - z = 0. \) (b) \( x^2 - y^2 - z^2 = -1. \)

9. (10 pts) Find the partial derivatives \( f_x \) and \( f_y \) (and \( f_z \)).

(a) \( f(x, y) = y \tan^{-1}(xy^2). \) (b) \( f(x, y, z) = \left(\frac{x}{y}\right)^2. \)

10. (16 pts) The surface \( S : z = \sqrt{6 - x^2 - y^2} \) is a hemisphere of radius \( \sqrt{6} \) centered at the origin.

(a) Find a vector parametrization of the line \( l_1 \) tangent to the curve of intersection of \( S \) with the plane \( x = 1 \) at the point \((1, 2, 1). \)

(b) Let \( C \) be the intersection of \( S \) with the plane \( z + y = 2. \) Determine the projection of \( C \) onto the \( xy \)-plane and find a vector parametrization for it.