
Important: 3 points will be deducted from each of Problems 4, 8 and 10, if you do not use Green's, divergence, and Stokes's Theorems to compute the corresponding integrals.

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1. (15 pts) Use spherical coordinates to find the mass of the ball $x^2 + y^2 + z^2 \leq 9$ if the density is given by
$$\lambda(x, y, z) = \frac{z^2}{z^2 + y^2 + z^2}.$$

2. (15 pts) Take Ω as the parallelogram bounded by $x - y = 0,\ x - y = \pi,\ x + y = 0,\ x + y = \frac{1}{2}\pi$.

   Evaluate $$\iint_\Omega (3x + y)\ dxdy.$$

3. (15 pts) Calculate the work done by the force $\mathbf{F}(x, y, z) = (2xy + z^2)\mathbf{i} + x^2\mathbf{j} + 2xz\mathbf{k}$ applied to a particle that moves in a straight line from the point (0,1,1) to the point (2,2,1).

4. (15 pts) Use Green's theorem to evaluate the line integral $\int_C (3xy + y^2)\ dx + (2xy + 4x^2)\ dy$, where C is the circle $(x - 1)^2 + (y + 2)^2 = 1$ traversed in a counterclockwise manner.

5. (15 pts) Find the surface area of the surface $3z = x^{3/2} + y^{3/2}$ with $0 \leq x \leq 1,\ 0 \leq y \leq x$.

6. (15 pts) Evaluate the surface integral.
$$\iint_S xyz\ d\sigma; \quad S \text{ the first octant part of the plane } x + y + z = 1.$$

7. (15 pts) Let S be the parallelogram given by
$$\mathbf{r}(u, v) = (u + v)\mathbf{i} + (u - v)\mathbf{j} + 2u\mathbf{k}; \quad 0 \leq u \leq 1,\ 0 \leq v \leq 1.$$
   Determine the flux of $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$ across S in the direction of the fundamental vector product.

8. (15 pts) Use the divergence theorem to compute the total flux of $\mathbf{v}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ out of the solid cylinder: $x^2 + y^2 \leq 4,\ 0 \leq z \leq 3$, including the top and base.

9. (15 pts) Evaluate $\int_C 4z\ dx - 2x\ dy + 2x\ dz$ where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y + 1$, traversed in a counterclockwise manner.

10. (15 pts) The upper half of the ellipsoid $x^2 + y^2 + 2z^2 = 2$ intersects the cylinder $x^2 + y^2 - y = 0$ in a curve C. Calculate the circulation of $\mathbf{v} = y^3\mathbf{i} + (xy + 3xy^2)\mathbf{j} + z^4\mathbf{k}$ around C in the *clockwise* manner by using Stokes's Theorem.