Calculus Homework Assignment 5

Class: __________________________

Student Number: __________________________

Name: __________________________

1. Identify the inflection points and local maxima and minima of the function

\[ y = x + \sin 2x, \quad \frac{-2\pi}{3} \leq x \leq \frac{2\pi}{3}. \]

Identify the intervals on which the function is concave up and concave down. [§4.4 #85]

2. Use the steps of the graphing procedure on page 208 to graph the equation

\[ y = |x^2 - 1|. \]

Include the coordinates of any local and absolute extreme points and inflection points. [§4.4 #45]

4. Design a poster You are designing a rectangular poster to contain 300 cm² of printing with a 10 cm margin at the top and bottom and a 5 cm margin at each side. What overall dimensions will minimize the amount of paper used? [§4.5 #11]

The area of the printing is \((x-20)(y-10) = 300\)

Consequently, \(y = \left(\frac{300}{x-20}\right) + 10\)

The area of the paper is

\[ A(x) = x \left(\frac{300}{x-20} + 10\right), \text{ where } x > 20 \]

\[ A'(x) = \frac{10(x-20)^2 - 6000}{(x-20)^2} = 0 \]

\[ x = 600 + 20 \approx 44.5 \]

\[ y \approx 22.5 \]

Width 22.5 \text{ cm}

Length 44.5 \text{ cm}

(Over Please)
5. **Shortest beam**
The 3 m wall shown here stands 8 m from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.  

\[
\frac{3}{x} = \frac{h}{x+8} \Rightarrow 3(x+8) = hx \Rightarrow h = 3 + \frac{24}{x}
\]

and \( L(x) = \sqrt{\frac{9}{x^2} + (x+8)^2} \) when \( x > 0 \).

Note that \( L(x) \) is minimized when \( f(x) \)

\[
= \left(3 + \frac{24}{x}\right)^2 + (x+8)^2 \text{ is minimized.}
\]

If \( f(x) = 0 \), then \( 2 \left(3 + \frac{24}{x}\right)(\frac{-24}{x^2}) + 2(x+8) = 0 \)

\[\Rightarrow (x+8)(1 - \frac{24}{x^2}) = 0\]

\[\Rightarrow x = -8 \text{ (not valid)} \text{ or } x = \frac{3\sqrt{72}}{2} \]

\[\Rightarrow L(\frac{3\sqrt{72}}{2}) \approx 15 \text{ m} \]

6. Find the most general antiderivative or indefinite integral. Check your answers by differentiation.

a. \[\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right) \, dx\]

b. \[\int \frac{1 + \cos 4t}{2} \, dt \] \[\text{[§4.7 #23, 47]}\]

\[\text{a) } \int \left(\frac{1}{x} - x^2 - \frac{1}{3}\right) \, dx = \int \left(\frac{1}{x} - x^2 - \frac{1}{3}\right) \, dx = \frac{x}{1} - \frac{x^3}{3} - \frac{1}{3} \Rightarrow C = \frac{1}{x} - \frac{x^3}{3} - \frac{1}{3} + C\]

\[\text{b) } \int \frac{1 + \cos 4t}{2} \, dt = \int \left(\frac{1}{2} + \frac{1}{2} \cos 4t\right) \, dt = \frac{1}{2}t + \frac{1}{2} \left(\frac{\sin 4t}{4}\right) + C = \frac{t}{2} + \frac{\sin 4t}{8} + C\]

7. Find the curve \( y = f(x) \) in the \( xy \)-plane that passes through the point \((9, 4)\) and whose slope at each point is \( 3\sqrt{x} \).

\( m = \frac{dy}{dx} = 3\sqrt{x} \Rightarrow \frac{dy}{dx} = 2x^{\frac{1}{2}} + C\)

at \((9, 4)\) we have \(4 = 2(9)^{\frac{1}{2}} + C\)

\[\Rightarrow C = -50\]

\[\Rightarrow y = 2x^{\frac{1}{2}} - 50\]

8. Use finite approximation to estimate the area under the graph of \( f(x) = \frac{1}{x} \) between \( x = 1 \) and \( x = 5 \) using

a. a lower sum with two rectangles of equal width.

b. an upper sum with four rectangles of equal width.

\[\text{[§5.1 #3(a) · (d)]}\]

\(\text{a) } \Delta x = \frac{5-1}{2} = 2 \quad \text{and} \quad x_2 = 1 + \Delta x = 3 \quad \text{or} \quad \sum_{i=1}^{\infty} \frac{1}{x_i} = 2 \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{16}{15}\)

\(\text{b) } \Delta x = \frac{5-1}{4} = 1 \quad \text{and} \quad x_2 = 1 + \Delta x = 2 \quad \text{or} \quad \sum_{i=2}^{\infty} \frac{1}{x_i} = 2 \left(\frac{1}{2} + \frac{1}{3}ight) + \frac{1}{4}\]

\[\sum_{i=0}^{\infty} \frac{1}{x_i} = |(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})| = \frac{25}{12}\]
1. \( y' = x + \sin 2x \Rightarrow y' = 1 + 2 \cos x \)

\[ y' = \begin{bmatrix}
-\frac{2 \pi}{3} & -\frac{\pi}{3} & \frac{\pi}{3} & \frac{2 \pi}{3}
\end{bmatrix} \]

⇒ the graph is rising on \((-\frac{\pi}{3}, \frac{\pi}{3})\), falling on \((\frac{2 \pi}{3}, \frac{\pi}{3})\) and \((\frac{\pi}{3}, \frac{2 \pi}{3})\)

⇒ local maxima are \(-\frac{2 \pi}{3} + \frac{\sqrt{3}}{2}\) at \(x = \frac{2 \pi}{3}\) and \(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\) at \(x = \frac{\pi}{3}\)

local minima are \(-\frac{2 \pi}{3} - \frac{\sqrt{3}}{2}\) at \(x = -\frac{\pi}{3}\) and \(\frac{2 \pi}{3} - \frac{\sqrt{3}}{2}\) at \(x = \frac{2 \pi}{3}\)

\( y'' = -4 \sin 2x \)

\[ y'' = \begin{bmatrix}
-\frac{2 \pi}{3} & -\frac{\pi}{3} & 0 & \frac{\pi}{3} & \frac{2 \pi}{3}
\end{bmatrix} \]

⇒ the graph is concave up on \((-\frac{\pi}{2}, 0)\) and \((\frac{\pi}{2}, \frac{3 \pi}{2})\)

concave down on \((-\frac{\pi}{3}, \frac{\pi}{3})\) and \((0, \frac{\pi}{2})\)

⇒ point of inflection at \((\frac{\pi}{2}, 0)\), \((0, 0)\), \((\frac{\pi}{3}, \frac{\pi}{3})\)
2. \( y = |x^2 - 1| = \begin{cases} x^2 - 1, & |x| \geq 1 \\ 1 - x^2, & |x| < 1 \end{cases} \), then

\[ y' = \begin{cases} 2x, & |x| > 1 \\ -2x, & |x| < 1 \end{cases} \text{ and } y'' = \begin{cases} 2, & |x| > 1 \\ -2, & |x| < 1 \end{cases} \]

The curve rises on \((-\infty, 0)\) and \((1, \infty)\) all falls on \((-\infty, -1)\) and \((0, 1)\).

There is a local maximum at \(x = 0\) and local minimum at \(x = \pm 1\).

The curve is concave up on \((-\infty, -1), (1, \infty)\) concave down on \((-1, 1)\).

There are no points of inflection because \(y\) is not differentiable at \(x = \pm 1\).